# MICRO-428: Metrology

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# MICRO-428: Metrology

## Week Ten: Electrical Metrology

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# Reference Books


📖 B. Razavi, *Design of Analog CMOS Integrated Circuits, McGraw Hill., 2017*

📖 C. Kittel, *Elementary Statistical Physics*, John Wiley, 1958

📖 E. Charbon, “*Imaging Sensors – ET 4390 Course Slides*”, Delft 2016

📖 S. Cova, “*Sensors Signals and Noise – Course Slides*”, Politecnico di Milano 2016 (Noise1, 2, 3, HPF1)
-> link: http://home.deib.polimi.it/cova/elet/lezioni/lezioni.htm

📖 I. Rech, “*Signal Recovery 2021-2022 – Course Slides*”, Politecnico di Milano (HPF1 – SR13)
-> link: https://rech.faculty.polimi.it/?page_id=235

# Week 9 Summary

S

9.0 Random Variables/2: Mean, 2nd Order Moment and Variance of all previously introduced distributions (8.2), plus Gamma: $Y \sim \mathrm{Gamma}(a, \lambda)$, and some of their corresponding salient properties

9.1 Random (or stochastic) Process RP as a collection of an infinite number of Random Variables. Concept of Ensemble of signals (= set of all possible sample functions).

PDF $f_X(x,t) \leftrightarrow$ CDF $F_X(x,t)$, expected value (mean) $E\{X(t)\}=m_X(t) = \overline{X(t)}$, variance $Var\{X(t)\}$

$$= E\left\{\left(X(t) - m_X(t)\right)^2\right\} = \sigma^2$$

$$\rightarrow C_{XX}(t_1, t_2), K_{XX}(t_1, t_2)\text{: auto}-\text{covariance/correlation}$$

$$\rightarrow C_{XY}(t_1, t_2), K_{XY}(t_1, t_2)\text{: cross}-\text{covariance/correlation}$$

# Week 9 Summary

S

9.1 Stationary RP: statistical properties do not change in time. Weaker form: Wide-Sense Stationary, constant mean + $K_{XX}(t, t+\tau) = K_{XX}(\tau)$

Ergodic random processes: statistical properties can be deduced from a single, sufficiently long, random sample: Ensemble average ↔ Time average

$$\overline{X(t)} = E\{X(t)\} = \langle X(t)\rangle, K_{XX}(\tau) = E\{X(t)\cdot X(t+\tau)\} = \mathcal{K}_{XX}(\tau)$$

9.2 Law of Large Numbers: as $n$ grows, the sample mean $\overline{X_n}$ converges to the true mean $\mu$

$$E\{\overline{X_n}\} = \mu, \mathrm{Var}\{\overline{X_n}\} = \frac{\sigma^2}{n}, \text{assuming i.i.d. } X_1, X_2 X_3 \ldots \text{ RVs}$$

Central Limit Theorem: Sum of a large number of i.i.d. random variables has an approximately Gaussian (normal) distribution

9.3 Elements of Estimation Theory: how do we use collected data to estimate unknown parameters of a distribution?

9.4 Accuracy (-> mean), Precision (-> spread), Resolution

# Outline

# 10.1.1 Charge

S

- Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field.
- Charge is measured in coulomb [unit: C] named after French physicist Charles-Augustin de Coulomb.
- The symbol $Q$ often denotes charge.
- Proton and electron have equal and opposite elementary (indivisible) charge = $1.602\ 176\ 634 \times 10^{-19}$ C (exact by definition of the coulomb, 20/05/2019)

  -> Experiments to determine the existence of fractional charges

  - Quarks: charge quantized into multiples of 1/3e (but cannot be seen as isolated particles)
  - Quasiparticles can also have fractional charges


Charles-Augustin de Coulomb

EN Wikipedia *Electric_charge*

# 10.1.1 Charge

S

- Conservation of Charge: Electric charge is a conserved property; the net charge of an isolated system, the amount of positive charge minus the amount of negative charge, cannot change.

-> Charge-current continuity equation: $\rho$ = charge density, **J** = current density, *I* = net current, *V* = volume of integration, *S* = closed surface $\partial V$, *Q* = charge contained within *V*

$$-\frac{d}{dt}\int_V \rho \, dV = \oiint_{\partial V} \mathbf{J}\cdot d\mathbf{S} = \int J \, dS \cos\theta = I$$

$$I = -\frac{dQ}{dt}, Q = \int_{t_1}^{t_2} I \, dt$$

- How to measure charge:
  - **Millikan's oil drop experiment (1909)**
  - Shot noise (analyse the noise of a current), Walter H. Schottky


Charles-Augustin de Coulomb

- Charge transported by a constant current of 1 A in 1 s:

$$1\,\mathrm{C} = 1\,\mathrm{A} \times 1\,\mathrm{s}$$

- Amount of excess charge on a capacitor of 1 F charged to 1 V

$$1\,\mathrm{C} = 1\,\mathrm{F} \times 1\,\mathrm{V}$$

EN Wikipedia *Electric_charge*

# 10.1.2 Current

S

- An electric current is a flow of charge.

- Current is the rate at which charge is flowing in a circuit. It is the amount of charges that pass through any point of the circuit per unit time.

$$\text{Steady: } I = \frac{Q}{t}, \text{ general: } I = {}^{dQ}\!/_{dt}$$

- The conventional symbol for current is *I*, which originates from the French phrase *intensité du courant* (current intensity)

- The ampere is the base unit of electric current in the International System of Units (SI). It is named after André-Marie Ampère (1775–1836), French mathematician and physicist, considered the father of electrodynamics.

André-Marie Ampère

EN Wikipedia *Electric_current*

# 10.1.2 Current

S

$$\text{Ohm's Law (linear materials, low frequency): } \mathrm{I} = {}^{V}\!/_{R}$$

$$\text{Ohmic (Joule's) heating: } P = IV = I^2R = {}^{V^2}\!/_{R}$$

- Microscopically, a current can be carried by a flow of electrons, of (electron) holes acting as positive carriers, of ions or other charged particles, etc. E.g. in a semiconductor: drift + diffusion

$$J = \sigma E + Dq\nabla n$$

- Higher frequencies: *skin effect* (higher current density towards the surface)

André-Marie Ampère

EN Wikipedia *Electric_current*

# 10.1.3 Voltage

S

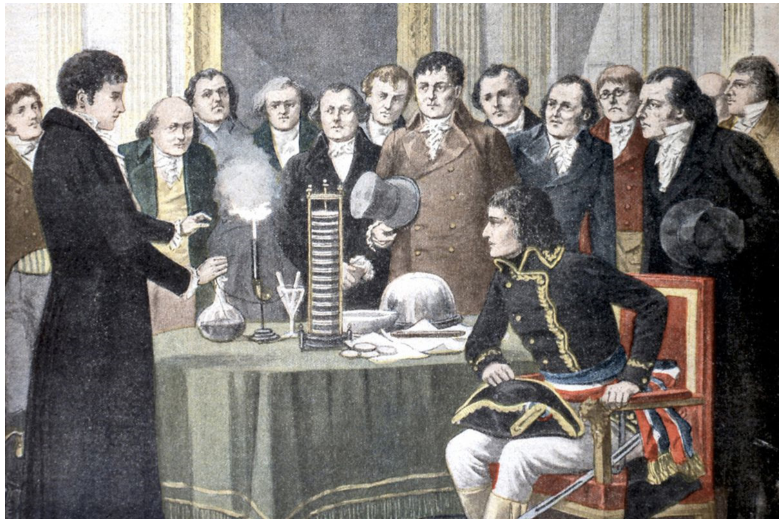
- Voltage, electric potential difference, electric pressure or electric tension is the difference in electric potential between two points.

- The difference in electric potential between two points (i.e., voltage) in a static electric field is defined as the work needed per unit of charge to move a test charge between the two points.

$$\Delta V_{AB} = V(x_B) - V(x_A) = -\int_{r_0}^{x_B} \vec{E} \cdot d\vec{l} - \left( \int_{r_0}^{x_A} \vec{E} \cdot d\vec{l} \right) =$$

$$= -\int_{x_A}^{x_B} \vec{E} \cdot d\vec{l}$$

- In the International System of Units, the derived unit for voltage is named volt ($V$). The volt is named in honour of the Italian physicist Alessandro Volta (1745–1827), who invented the voltaic pile, possibly the first chemical battery.



Alessandro Volta amazes Napoleon with his battery.

Voltage across an inductor:

$$\Delta V = -L \frac{dI}{dt}$$

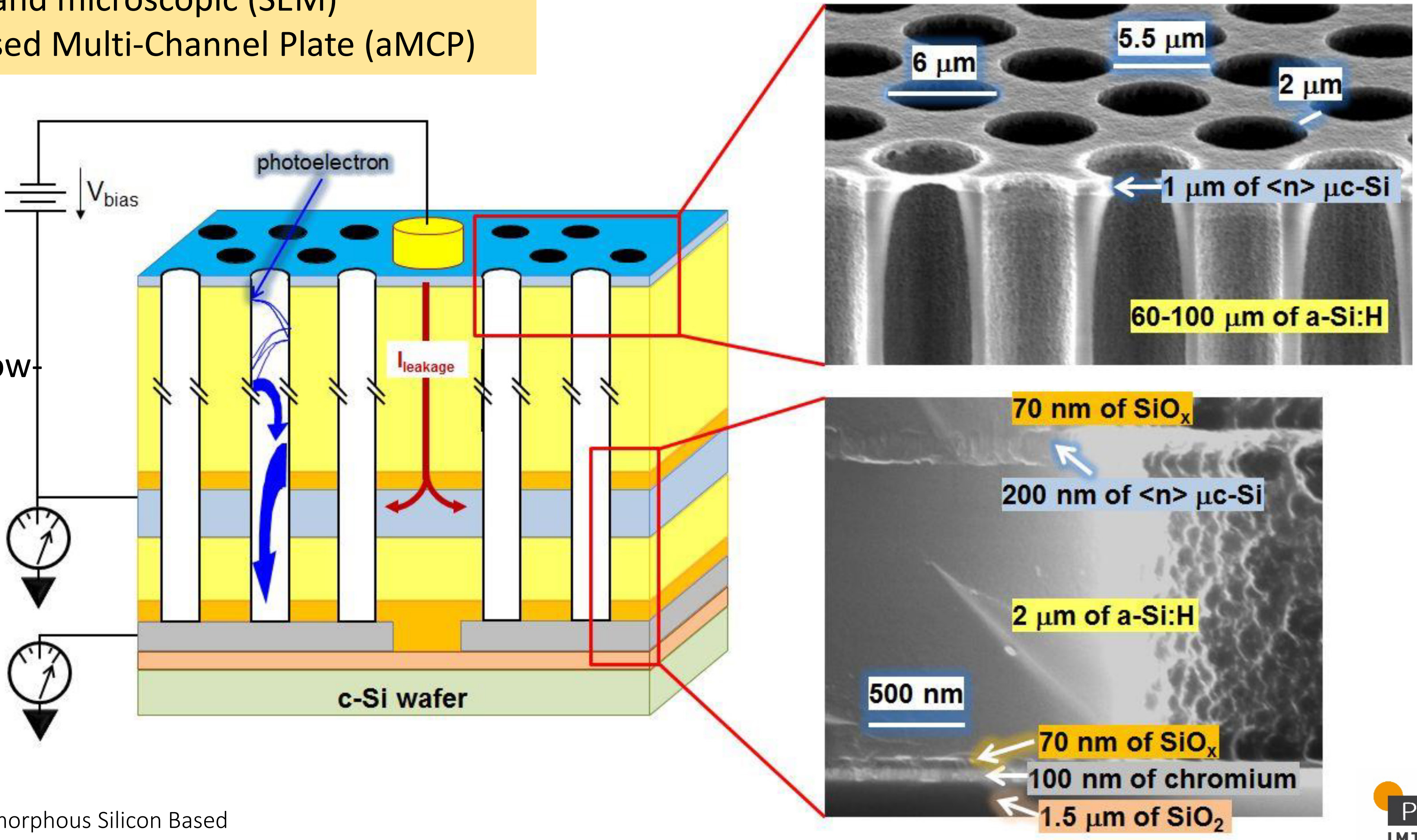
EN Wikipedia *Voltage*

# Back to the Basics – Example Application

Rendering and microscopic (SEM)
view of an a-Si:H based Multi-Channel Plate (aMCP)

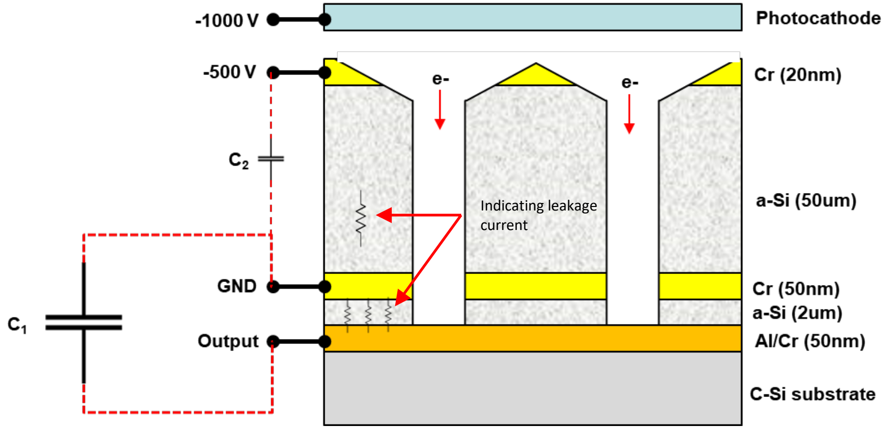
Problem: estimate aMCP capacitance to optimize follow-up low noise amplifier

*Q*

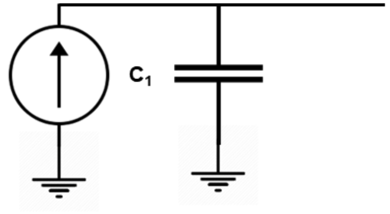



Frey, S., et al. "Characterization of Amorphous Silicon Based Microchannel Plates with High Aspect Ratio." 2019 IEEE NSS/MIC

# Back to the Basics – Example Application




Cross-section view of the aMCP channels



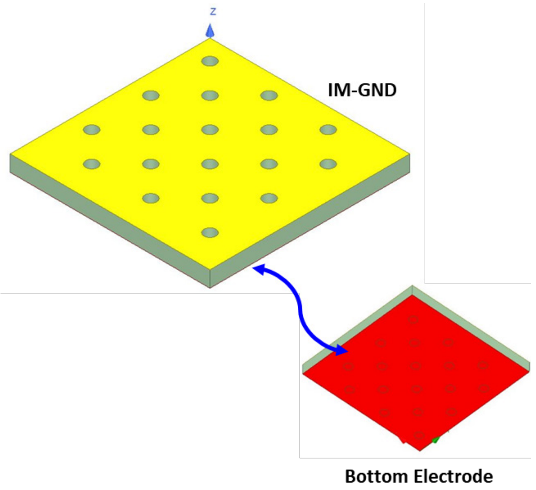

Simplified electrical model of AMCPs

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d}$$

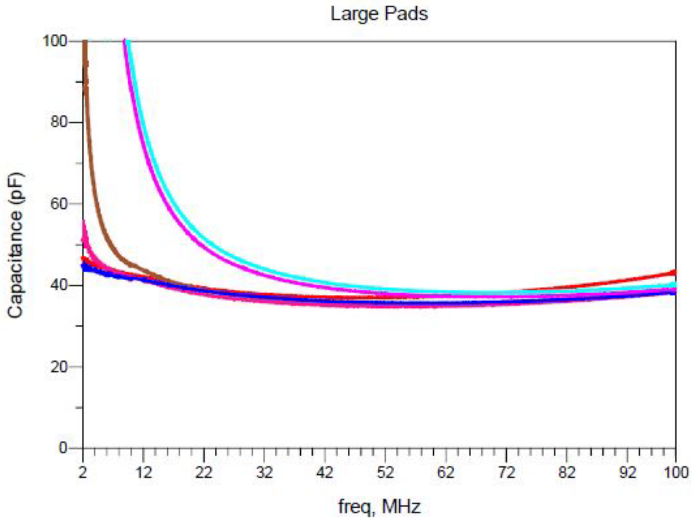
$\varepsilon_r$ → ~12 for a-Si:H*

Estimation of AMCP capacitance via parallel plate capacitor (~960 × 960 μm²) → **$C_1$=~45.5 pF**

vs. ANSYS simulation → **$C_1$=49.37 pF**




vs. Experimental results (notice a few broken channels)




Frey, S., et al. "Characterization of Amorphous Silicon Based Microchannel Plates with High Aspect Ratio." 2019 IEEE NSS/MIC

# Outline

# 10.2 Sample Statistics – Wide-sense Stationary Noise (WSS)

- Recap: Noise is generally modeled as a random process *n*(*t*), i.e. a collection of random variables, one for each time instant *t* in interval ]-∞,+∞[
- Each Random Variable (RV) has a probability density function *p*(*n,t*)
- In a stationary noise, *p*(*n,t*) is invariant in time
- Weaker form: in wide-sense stationary noise (WSS)…

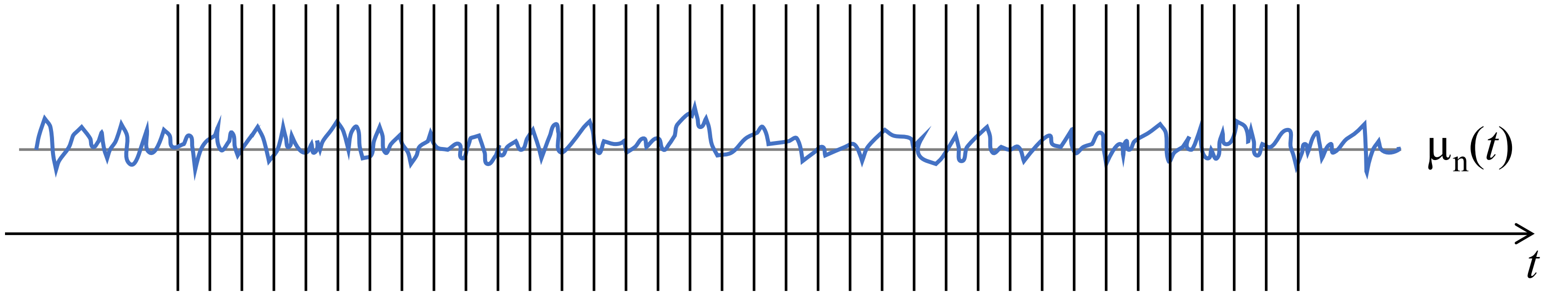
See also Section 9.1.1 & 9.1.2

(same message but seen from a noise perspective and adapted notation)

Mean : $E\{n(t)\} = \mu_n(t) = \mu_n(t+\tau) \quad \text{for all } \tau$

Autocorrelation: $E\{n(t_1) \cdot n(t_2)\} = K_{nn}(t_1, t_2) = K_{nn}(t_1+\tau, t_2+\tau) \quad \text{for all } \tau$

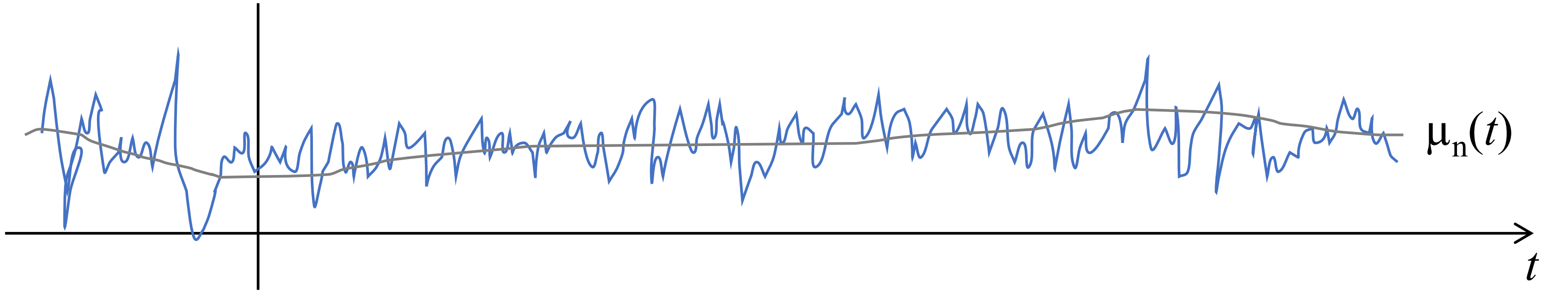
(the autocorrelation function only depends on the time difference, but not on the absolute position in time)




📖 E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

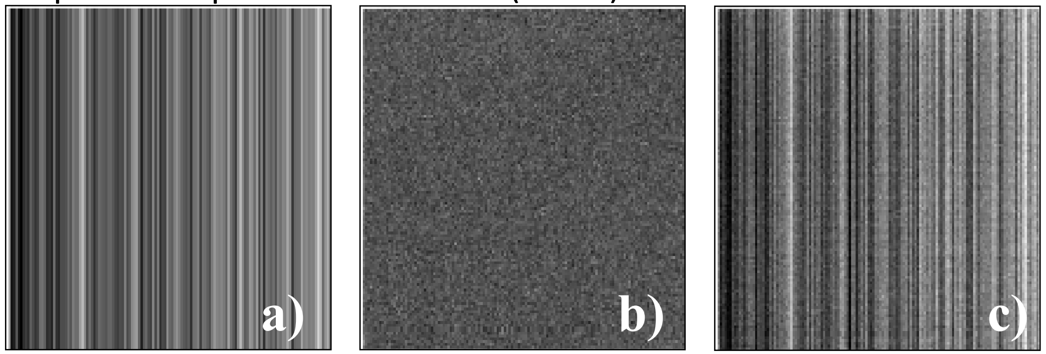
# 10.2 Time vs. Space

- In a non-stationary noise, $p(n,t)$ is **variant** in time




- A random process may be time-invariant but variant in space (x and/or y)

Example: fix-pattern noise (FPN)

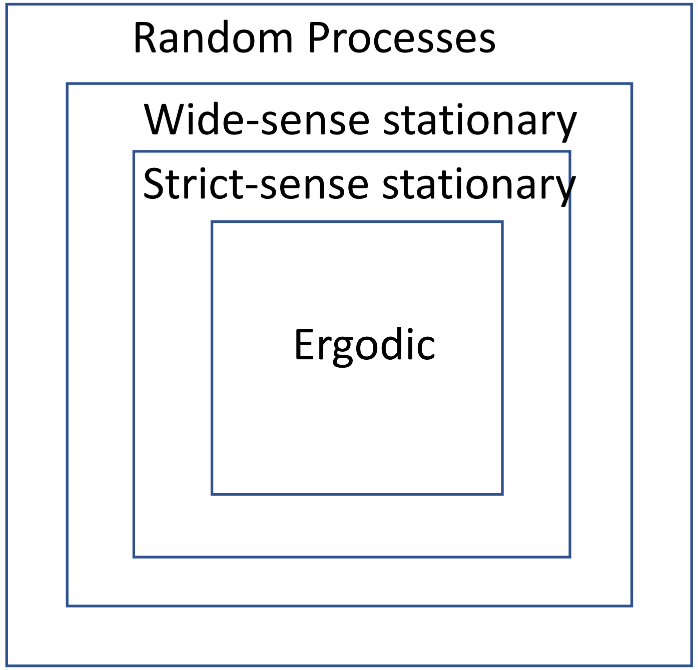



Pictures from J Goy PhD (INP Grenoble, 2002)

a) FPN in x

b) FPN in y

c) FPN in x and y




E. Charbon, “Imaging Sensors – ET 4390 Course Slides”, Delft 2016

# 10.2 Noise Source Characterization

Recap: $k$-Moments of RV $n$ **at time $t$** (but omitting the dependence)

$$m_k = E\{n^k\} = \int_{-\infty}^{\infty} n^k p(n)\, dn$$

Example:

$$m_0 = 1$$
$$m_1 = E\{n\} = \int_{-\infty}^{\infty} n\, p(n)\, dn = \mu_n$$
$$m_2 = E\{n^2\} = \int_{-\infty}^{\infty} n^2 p(n)\, dn$$

$$\sigma^2 = E\{n^2\} - \mu_n^2$$

Note: at each point in time t the RV n(t) will have exactly the same statistical properties if it is i.i.d.

See also Section 8.3

i.i.d. = independent and identically distributed Random Variables, have the same PDF and are all mutually independent

-> a collection of i.i.d. RVs implies a WSS process, but not vice versa (sufficient but not necessary condition).

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

# 10.2 Noise Power

Compute the autocorrelation of $n$ at time $t_1$:

$$K_{nn}(t_1, t_1 + \tau) = E\{n(t_1) \cdot n(t_1 + \tau)\}$$

$$\text{call } n(t_1) = n_1 \text{ and } n(t_1 + \tau) = n_2$$

See also Section 9.1.1 & 9.1.2

$$\rightarrow K_{nn}(t_1, t_1 + \tau) = \int_{-\infty}^{\infty} n_1\, p(n_1)\, n_2\, p(n_2)\, dn_1\, dn_2$$

If $n(t)$ is a WSS Process, then:

$$K_{nn}(t_1, t_1 + \tau) = K_{nn}(\tau)$$

i.e. the autocorrelation is independent of $t_1$. Follows that:

$$K_{nn}(0) = E\{n^2(t)\} = \overline{n^2(t)} \quad (= \sigma_n^2 \text{ assuming 0 mean: } \mu_n = 0)$$

This is the noise power!

📖 E. Charbon, “Imaging Sensors – ET 4390 Course Slides”, Delft 2016

# 10.2 Note on the Power of a Signal

- The average power $P$ of a signal $x(t)$ is defined as:

$$P = \lim_{T \to \infty} \frac{1}{T} \int_0^T |x(t)|^2 dt$$

*NB: the power can be the actual physical power, or more in general, the squared value of the signal*

(think also in terms of a hypothetical voltage source which followed $x(t)$ applied to the terminals of a 1 Ohm resistor -> instantaneous power dissipated in that resistor would be $|x(t)|^2$ Watt).

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016 ; EN Wikipedia *Spectral_density*

# 10.2 Power Spectral Density & Noise Power

- Concept of PSD (power spectral density), which describes how the power of a signal or time series is distributed over frequency; *total area under the PSD = total power*.

  The concept of PSD is particularly relevant in the case of noise because…

- Since it is a collection of RVs, noise (whether a current or a charge or a voltage) cannot be represented in the same way as a deterministic signal. Only the power (spectral density) is a valid representation of it, because that's what we are ultimately interested in.

- It can be shown that the noise power has a spectrum (Wiener–Khinchin theorem):

$$S_n(\omega) = \Im\{K_{nn}(\tau)\}\,, \text{with } P = K_{nn}\,(0) \propto \int_{-\infty}^{+\infty} S_n(\omega)\,d\omega$$

assuming WSS noise. For non-stationary noise, instead:

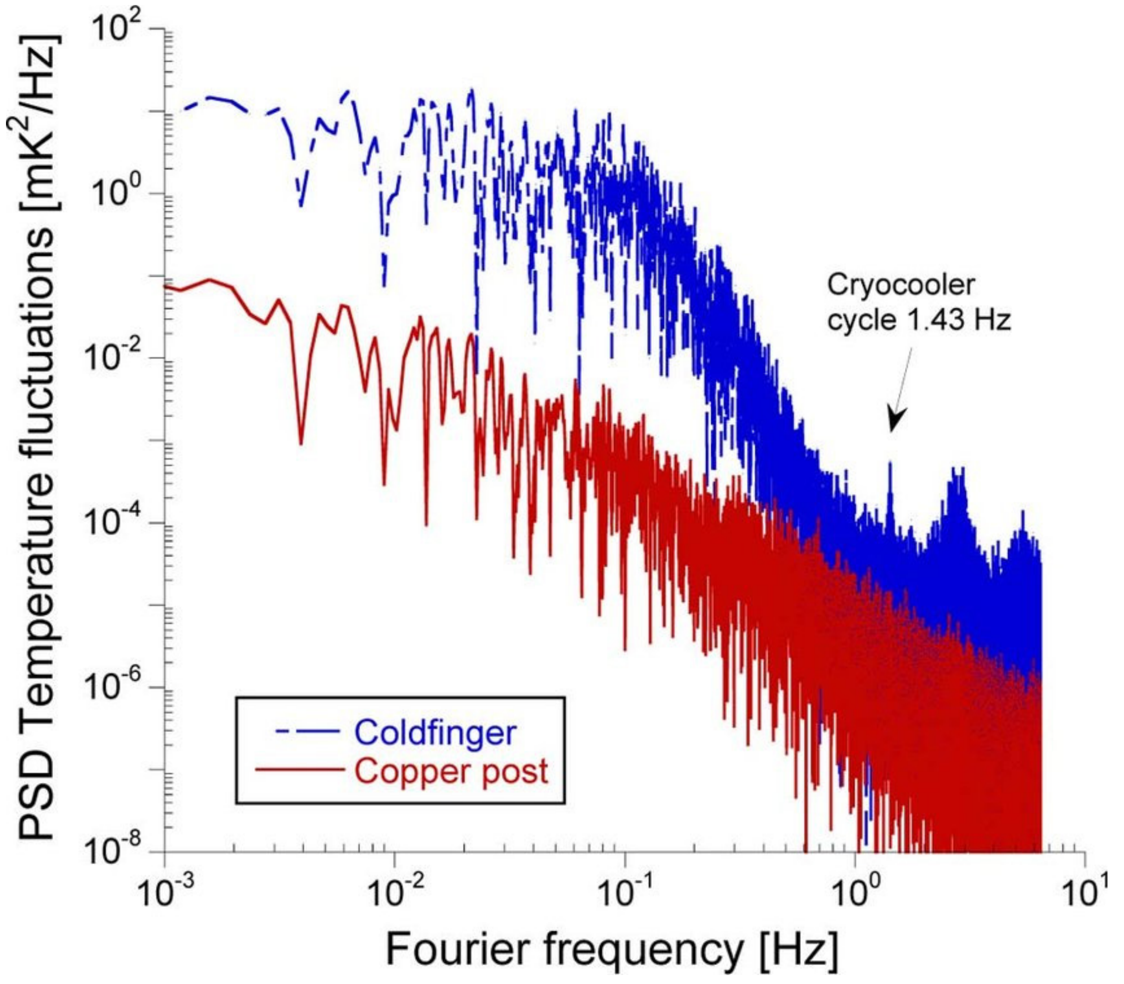
$$S_n(t_1,\omega) = \Im\{K_{nn}(t_1,\tau)\}$$

Ex

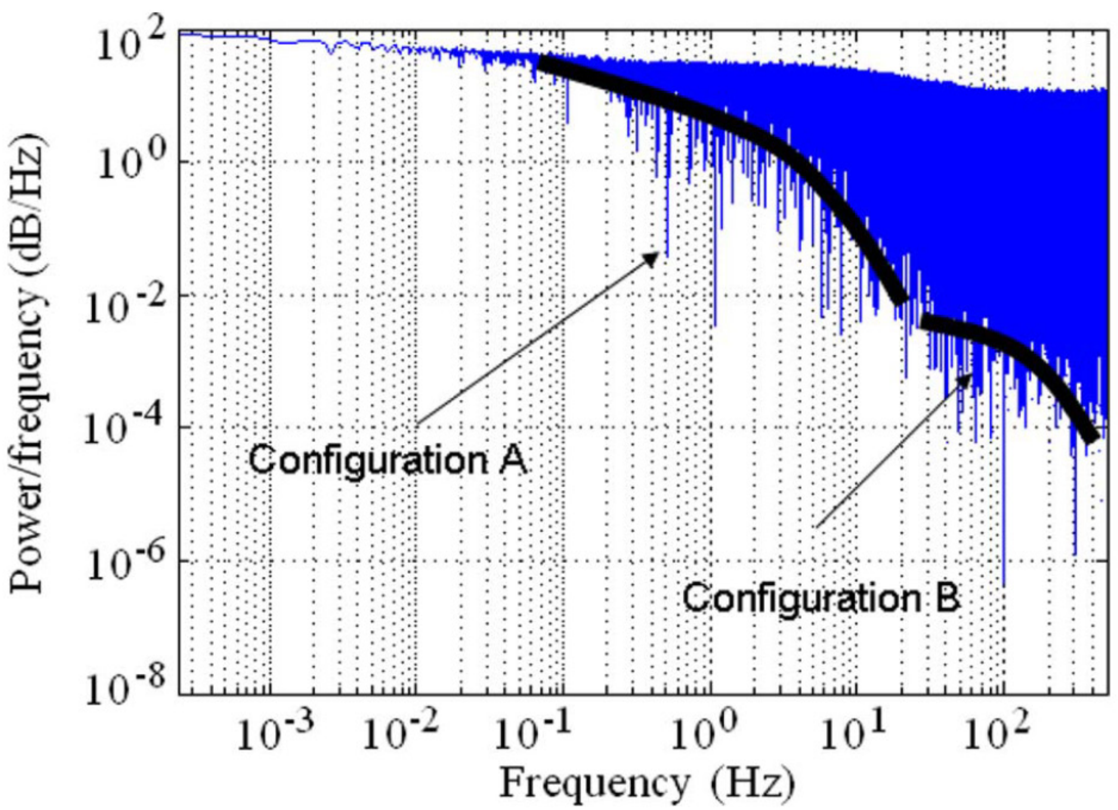
Note: average noise powers of *uncorrelated* noise sources add up!

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016 ; EN Wikipedia *Spectral_density* ; B. Razavi, "Design of Analog CMOS Integrated Circuits", 7.1.1, McGrawHill 2017

# 10.2 Power Spectral Density Examples



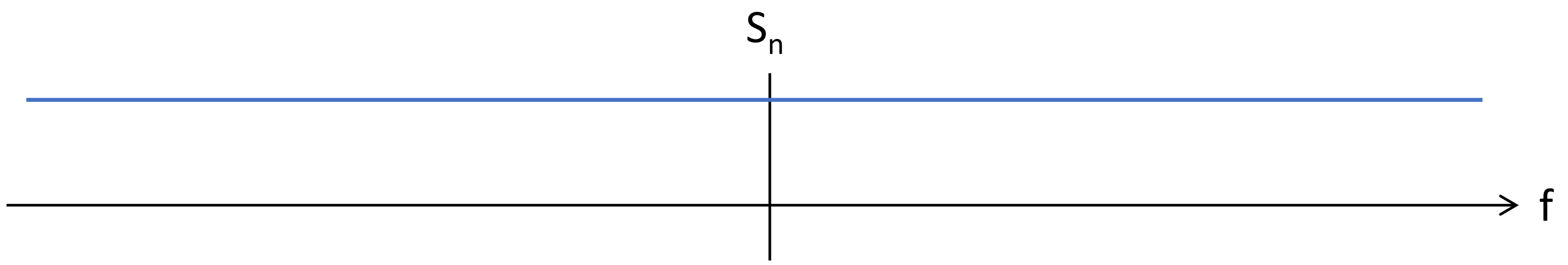

RTS PSD example (slide 58)




📖 **J. G. Hartnett & N. R. Nand**, IEEE TMTT(58), 2010. M. A. Karami et al., *Random Telegraph Signal in Single-Photon Avalanche Diodes*, International Image Sensor Workshop, Bergen, 2009

# 10.2 Noise Power Spectrum – White Noise

- Common type of noise **PSD (power spectral density)** is white noise.
- Displays same value at all frequencies. *Q*
- White noise does not exist strictly speaking since total power carried by noise cannot be infinite.
- Noise spectrum that is flat *in the band of interest* is usually called white.




B. Razavi, "Design of Analog CMOS Integrated Circuits", McGrawHill 2017

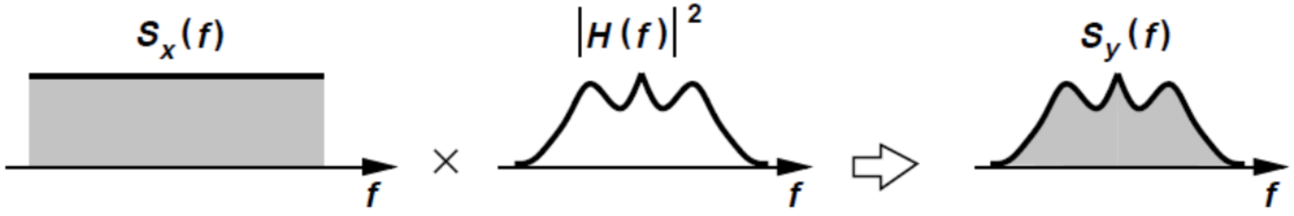
# 10.2 Noise Power Spectrum – Theorem

The PSD is a powerful tool in analyzing the effect of noise in circuits, especially in conjunction with the following Theorem:

- If a signal with spectrum $S_x(f)$, i.e. input PSD, is applied to a linear time-invariant (LTI) system with transfer function $H(s)$, then the output spectrum $S_Y(f)$, i.e. output PSD, is given by:

$$S_Y(f) = S_x(f)|H(f)|^2 \qquad \text{where } H(f) = H(s = j2\pi f)$$

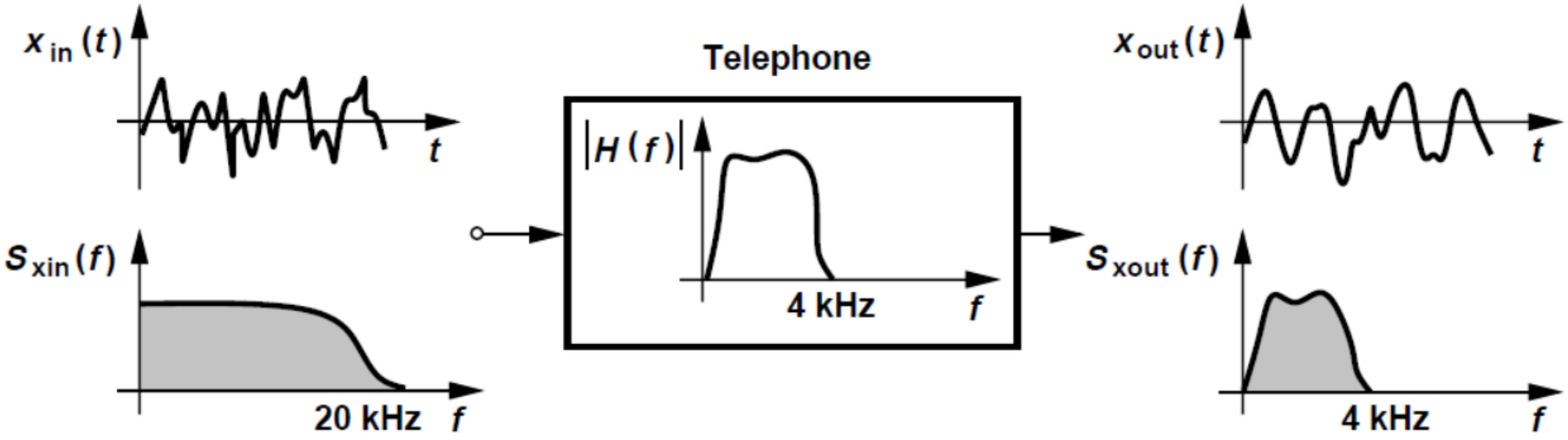
- In other words, the spectrum of the signal is shaped by the transfer function of the system.




B. Razavi, "Design of Analog CMOS Integrated Circuits", 7.1.1, McGrawHill 2017

# 10.2 Noise Power Spectrum – Theorem Example

- Regular [*old…*] telephones have a bandwidth of approximately 4 kHz and suppress higher frequency components in caller's voice.




- Due to limited bandwidth, $x_{out}(t)$ exhibits slower changes than $x_{in}(t)$, in which case it can be difficult to recognize the caller's voice.

B. Razavi, "Design of Analog CMOS Integrated Circuits", McGrawHill 2017

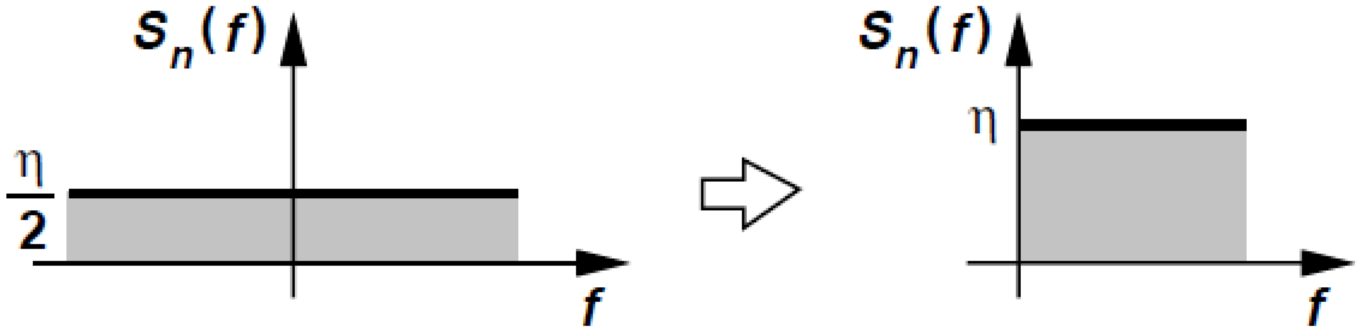
# 10.2 Noise Power Spectrum – Bilateral vs. Unilateral Spectra

- $S_n(f)$ is an even function of $f$ for real $n(t)$.

- PSD (power spectral density) can be specified as one-sided (only positive frequencies; typ. engineering) or two-sided functions (positive and negative frequencies; 2x smaller; typ. physics):

negative frequency part of the spectrum is folded around the vertical axis and added to the positive frequency part.

- Example: two sided white spectrum can be folded around the vertical axis to give a one sided white spectrum:




B. Razavi, “Design of Analog CMOS Integrated Circuits”, McGrawHill 2017

# Noise Analysis Procedure

- Identify the sources of noise, determine each spectrum
- Find the transfer function from each noise source to the output (as if the source were a deterministic signal)
- Utilize the theorem $S_Y(f) = S_x(f)|H(f)|^2$ to calculate the output noise spectrum contributed by each noise source. (The input signal is set to zero.)
- Add all of the output spectra, paying attention to correlated and uncorrelated sources

→ Output noise spectrum, to be integrated from -∞ to +∞ to obtain the total output noise

→ We need the noise representation of various sources…

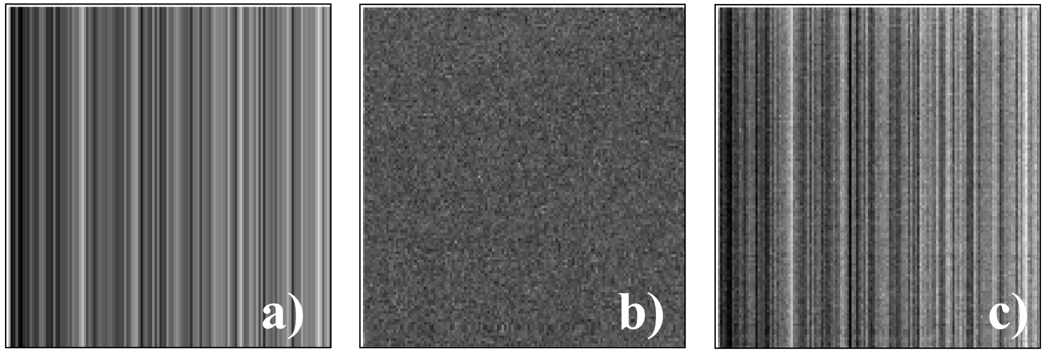
B. Razavi, "Design of Analog CMOS Integrated Circuits", 7.1.5, McGrawHill 2017

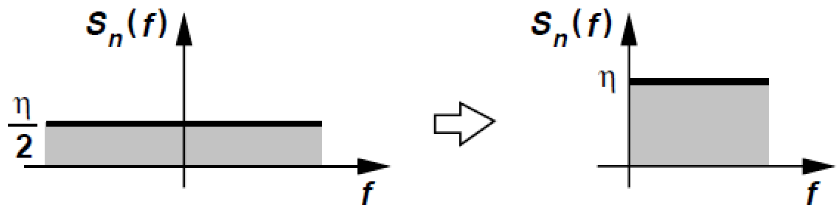
# Take-home Messages/W4-1

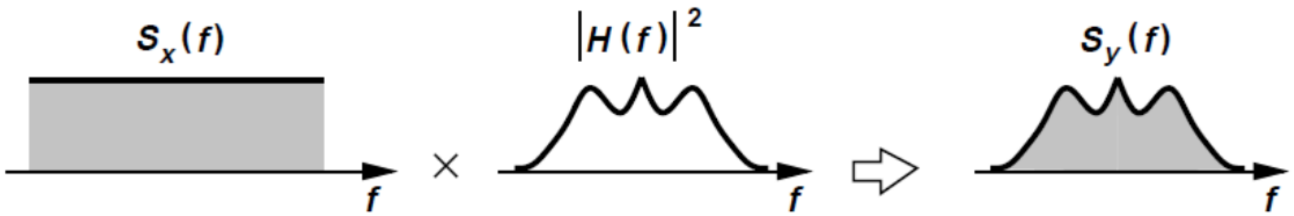
- *Noise modelled as a Random Process:*
  - Time vs. Space
  - Wide-sense Stationary Noise (WSS)
  - Noise Source Characterization
    - *n*-Moments
    - Noise Power and Noise Power Spectrum ->

      Power Spectral Density (PSD)
    - White Noise, Bilateral vs. Unilateral
    - LTI (linear time-invariant) systems

      & noise analysis procedure



# Outline

# 10.3 Noise Sources

10.3.1 Thermal Noise

10.3.2 kTC Noise

10.3.3 Shot Noise

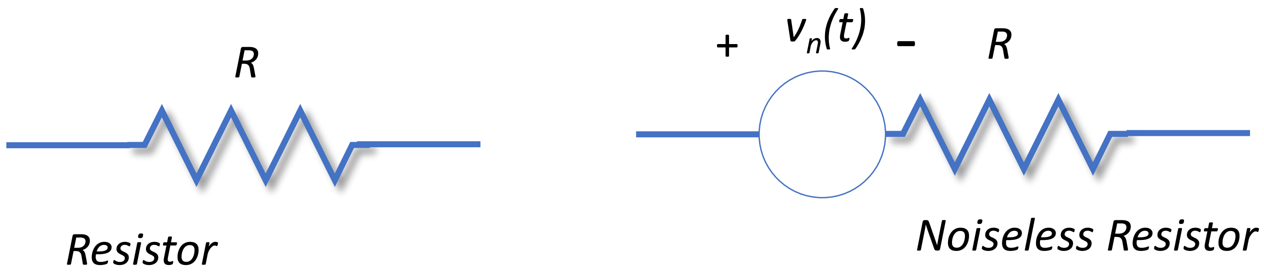
10.3.4 Flicker (1/f) Noise

10.3.5 RTS Noise

# 10.3.1 Thermal Noise

- Thermal noise is observed in any system having thermal losses and is caused by thermal agitation of charge carriers.
- An example of thermal noise can be in resistors. Random motion of electrons in a resistor induces fluctuations in the voltage measured across it even though the average current is zero.
- Thermal noise is also called as Johnson-Nyquist Noise.
- Thermal noise on a resistor can be modeled by a series voltage source $v_n(t)$.

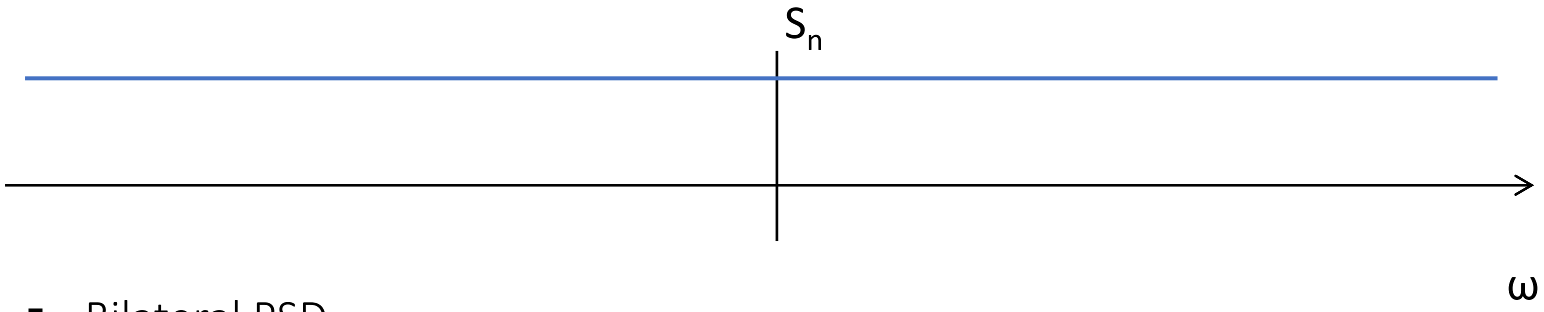
Ex




C. Kittel, *Elementary Statistical Physics*, John Wiley, 1958 ; E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006 -> K. B. Klaassen, Electronic measurement and instrumentation, Cambridge University Press, 1996.

# 10.3.1 Thermal Noise

- Thermal noise in frequency domain:

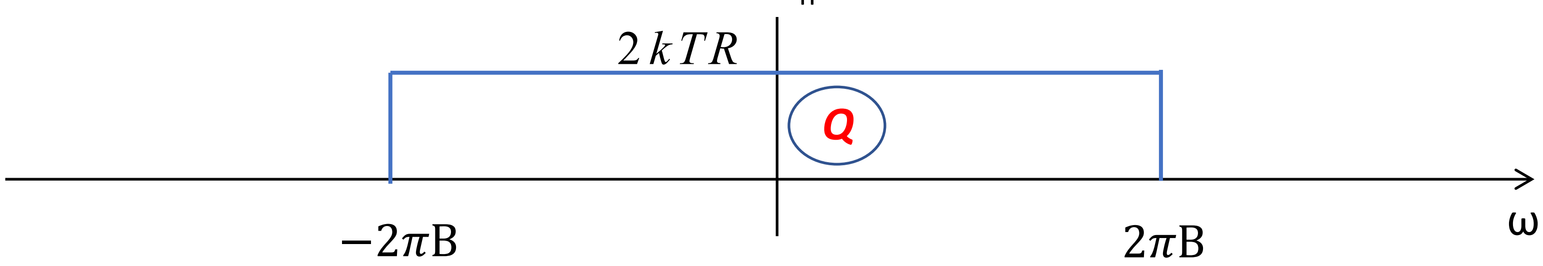



See also slide 18

$$K_{nn}(\tau) = 2kTR\delta(\tau)$$

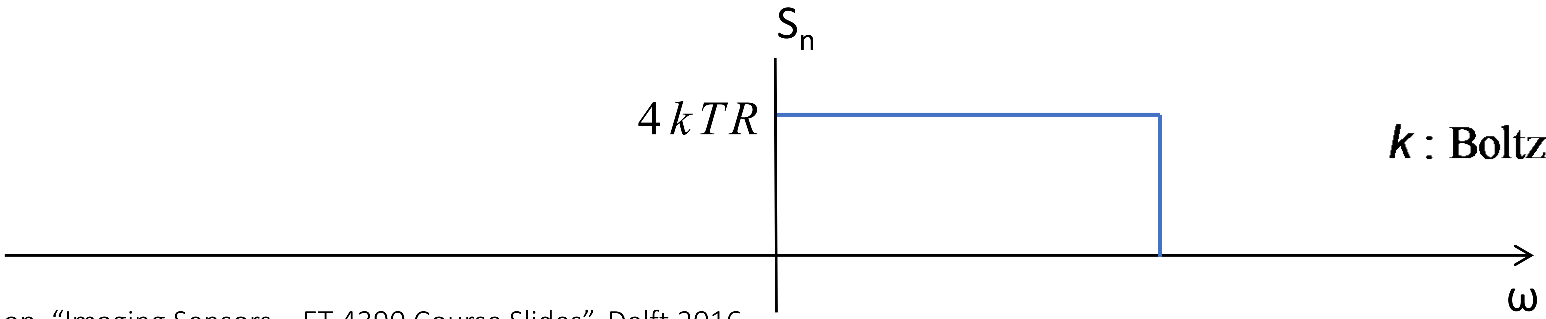
$$\rightarrow \overline{v^2(t)} = \sigma_n^2 = K_{nn}(0) = \infty!$$

- Bilateral PSD




$$\overline{v^2(t)} = \sigma_n^2 = 4kTR{\cdot}B$$

- Unilateral PSD



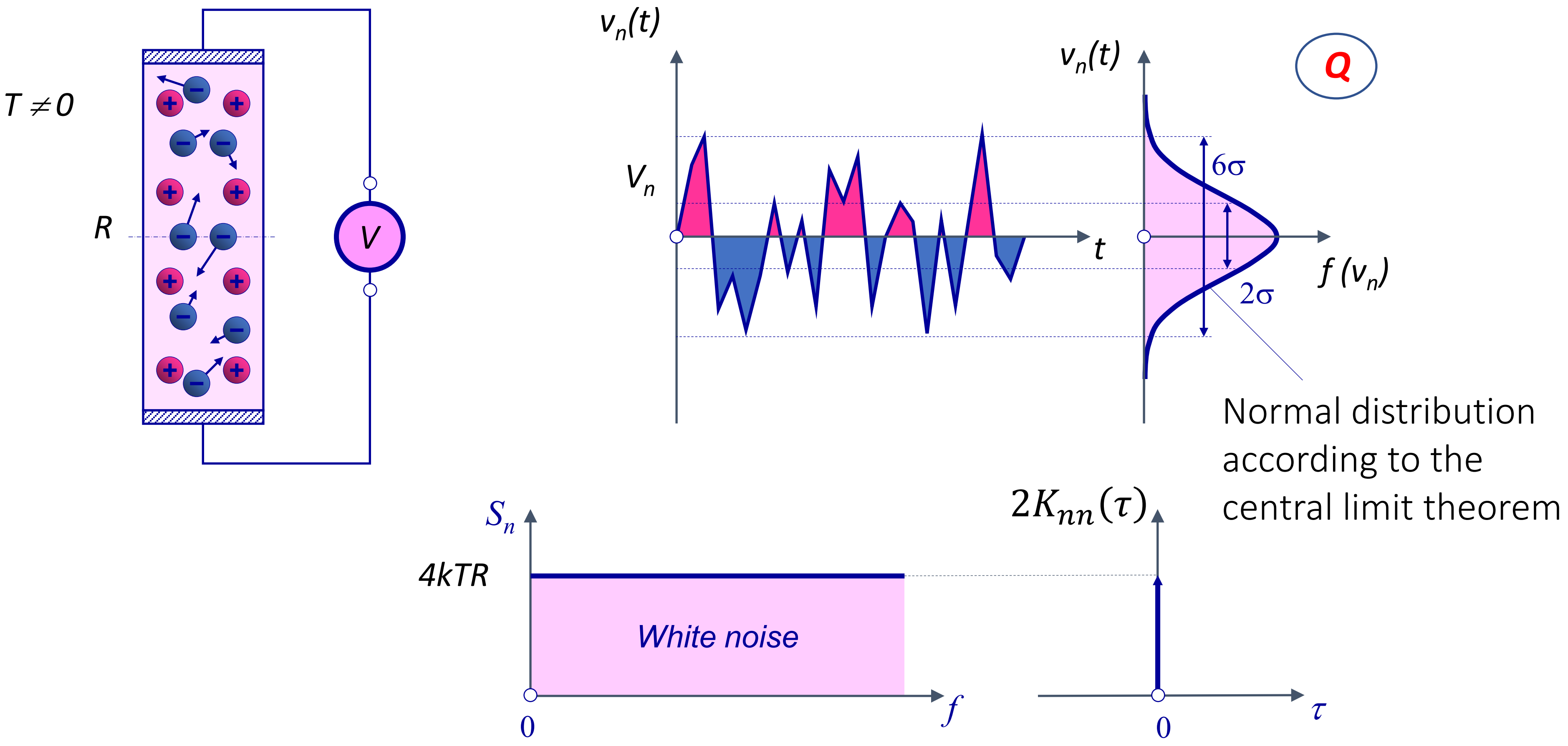

$$\sqrt{4kT} = 0.13nV/\sqrt{Hz}$$

$k$ : Boltzmann's constant ($=1.38 \times 10^{-23}$ J/K)

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

# 10.3.1 Thermal Noise

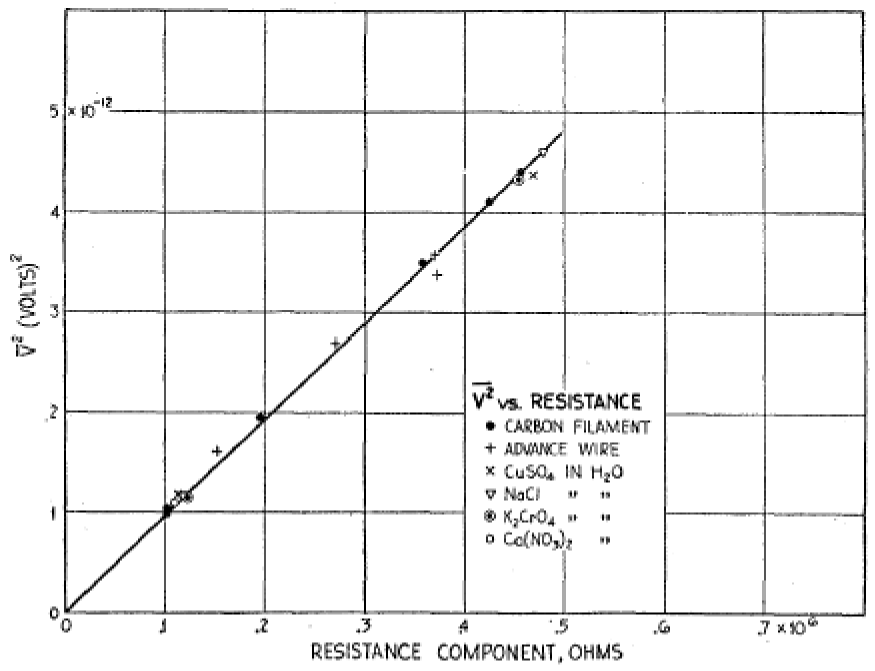
- Thermal noise of a Resistor

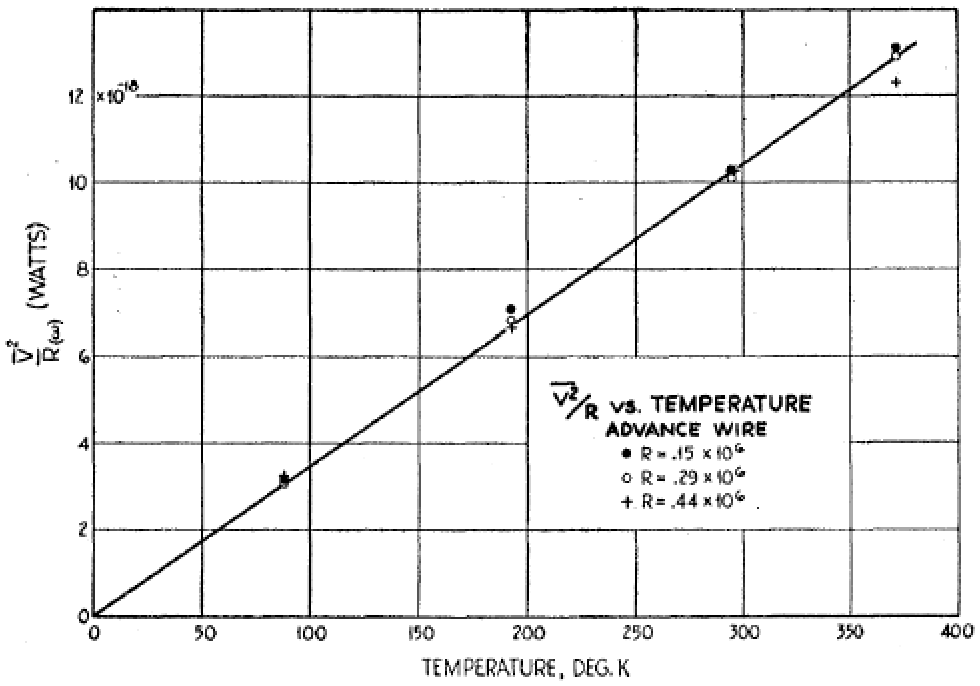



E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006

# 10.3.1 Thermal Noise (Johnson Experiment)

Thermal Noise of a *Resistor*




The thermal noise PSD is:

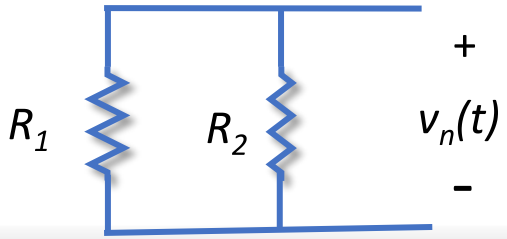
- Proportional to the resistance
- Proportional to the temperature

J.B. Johnson, "Thermal Agitation of Electricity in Conductors, Physical Review, vol. 32, pp. 97-109, 1928

# 10.3.1 Thermal Noise - Summary

- Only resistance creates thermal noise.
- Ideal capacitors and inductors do not generate any thermal noise.
- However, they do accumulate noise generated by other sources -> see kT/C noise....
- A noise source with a power spectrum that is flat and has a infinite bandwidth is called white noise.
- In practice, the bandwidth is never infinite but always limited.
- Since noise is a random quantity, the polarity used for the voltage or current source is unimportant.

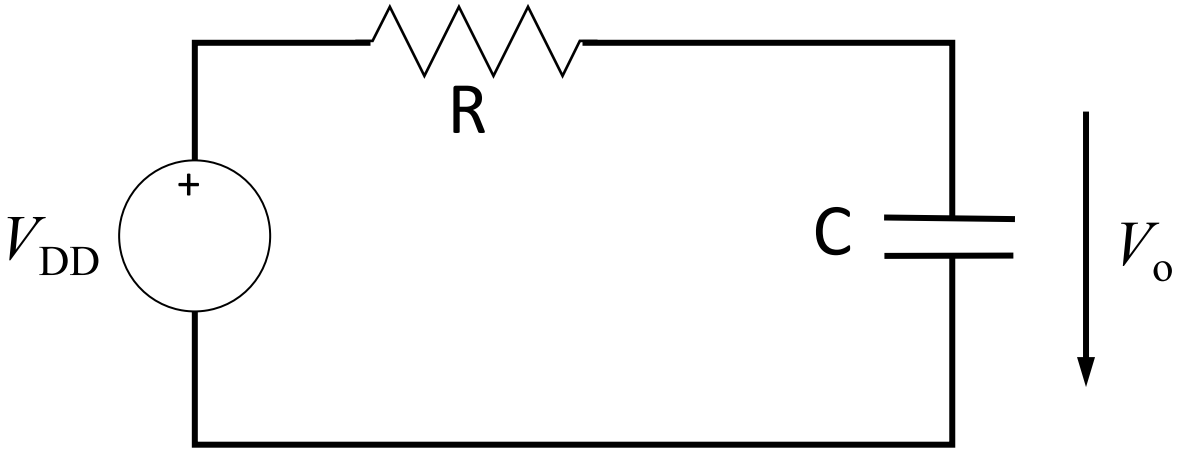
**Q** How to reduce it?




**Ex**

# 10.3.2 kTC Noise – Thermal Noise on Capacitors

- An ideal switch goes from closed (zero resistance) to open (infinite resistance) in zero time and hence would not create kTC noise.
- When a real switch is opened, it must have a resistance (even for a short amount of time):




RC circuit!

$$\overline{V_0^2(\omega)} = \left| \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \right|^2 \overline{v^2(\omega)}$$

→ Parseval

$$\overline{V_0^2(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{1}{1 + j\omega CR} \right|^2 \cdot \frac{4kTR}{2} \, d\omega$$

Parseval theorem:

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

E. Charbon, “Imaging Sensors – ET 4390 Course Slides”, Delft 2016

# 10.3.2 kTC Noise

Finally:

$$\overline{V_0^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1}{1+j\omega CR}\right|^2 \cdot \frac{4kTR}{2}\,d\omega$$

$$\overline{V_0^2} = \frac{kTR}{\pi}\int_{-\infty}^{\infty}\frac{1}{1+\omega^2C^2R^2}d\omega$$

$$\overline{V_0^2} = \frac{kTR}{CR\pi}\int_{-\infty}^{\infty}\frac{1}{1+x^2}dx = \frac{kTR}{CR\pi}[\arctan(x)]_{-\infty}^{\infty} = \frac{kTR}{CR\pi}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \frac{kT}{C}$$

Q

So, no matter how short time the resistance is in place (nor its value), this noise will appear! It is entirely due to thermal noise in the resistor.

Ex

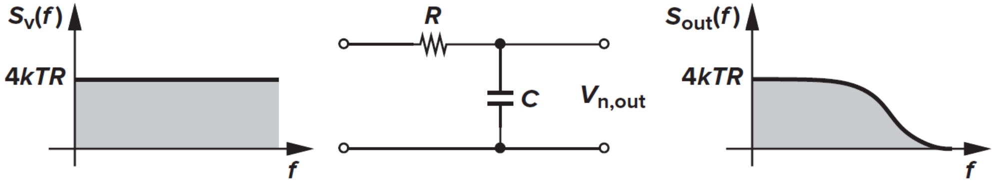


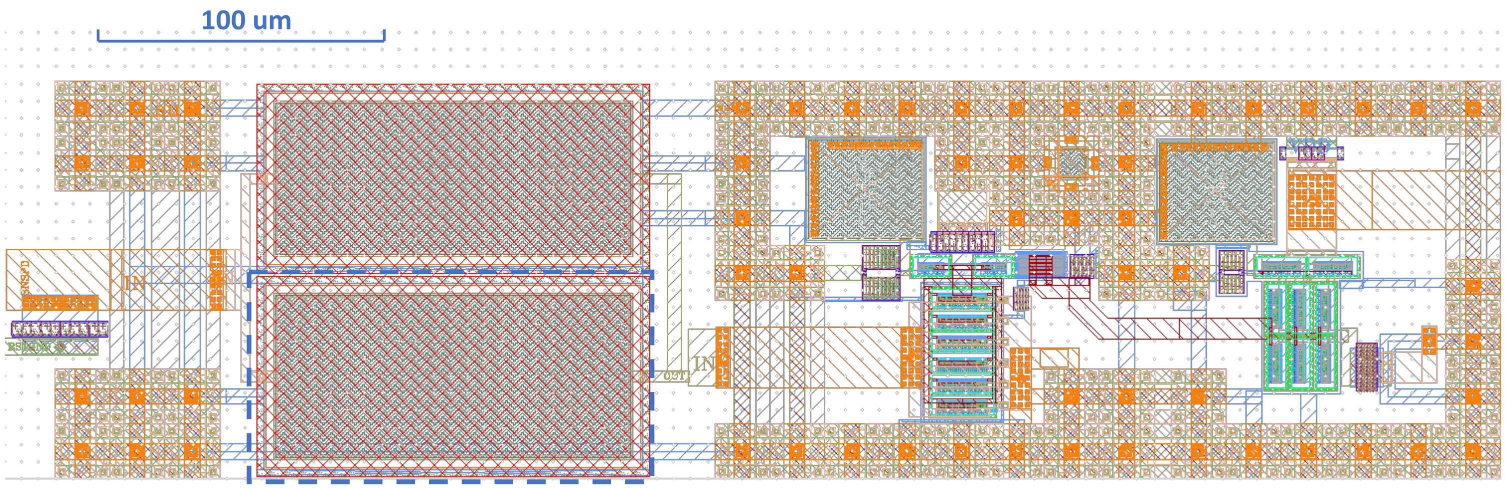

**Figure 7.16** Noise spectrum shaping by a low-pass filter.

RC circuit -> low-pass noise shaping!

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

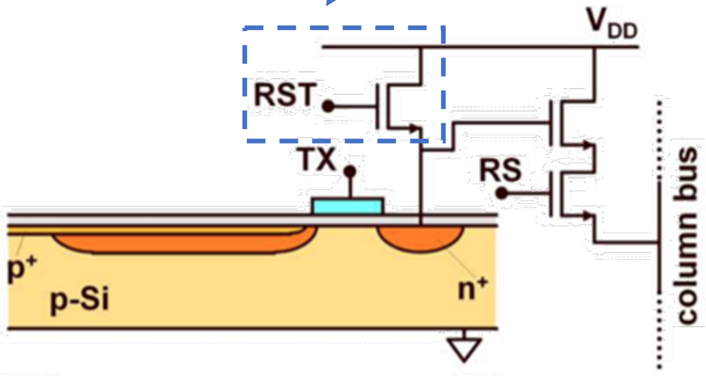
# 10.3.2 kTC Noise

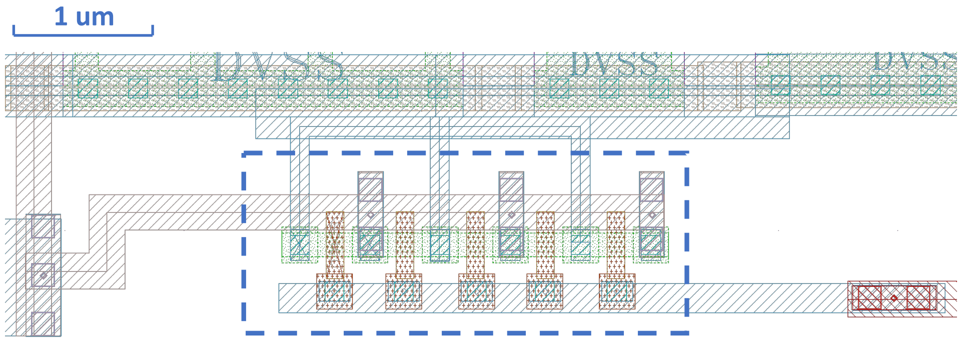
*Reduce C*? -> Microelectronics: not easy to increase the capacitance (area increase limited, especially in advanced processes).




Metal-insulator-Metal capacitor layout

Reset switch (Transistor) schematic




Reset switch (Transistor) layout

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

# 10.3.3 Shot (or Poisson) Noise

- Shot noise (derived by Schottky, 1918) results from the fact that the current is *not a continuous flow* but the *sum of discrete pulses*, each corresponding to the transfer of an electron through the conductor.
- Also in photon counting devices (associated to the particle nature of light)!
- May be dominant when few particles (charge carriers, photons) are involved -> large statistical fluctuations **Q**

*Example (electronics):*

- Generated with current flow through a potential barrier, e.g. in p-n junctions
- *Random* sequence of many *independent* pulses, i.e. *<< shots >>* due to single electrons that swiftly cross the junction depletion layer
  - Random fluctuations of current since *generation* and *recombination* is random
  - Happens in *semiconductor devices* and *vacuum tubes*

S. Cova, "Sensors Signals and Noise, Course Slides", Politecnico Milano 2016; M. de Jong, https://www.lorentz.leidenuniv.nl/beenakker/beenakkr/mesoscopics/topics/noise/noise.html

# 10.3.3 Shot Noise

- Assume pulses of rate $p$, charge $q$, and very short duration $T_h$ (shorter than transition times in circuits – ideally Dirac pulses): the *shot* current has mean value

Ex

$$I = {}^{dQ}/_{dt} = p \cdot q$$

- Shot current has fast fluctuations around the mean, called shot noise (or Schottky noise)

- It can be shown that the *Shot noise* has constant spectral density (bilateral formulation):

$$S_j(\omega) \cong qI \qquad \text{for } \omega \ll 2\pi 10\ GHz$$
$$K_{nn}(\tau) \cong qI\delta(\tau) \qquad \text{for } \tau \gg 100\ ps$$

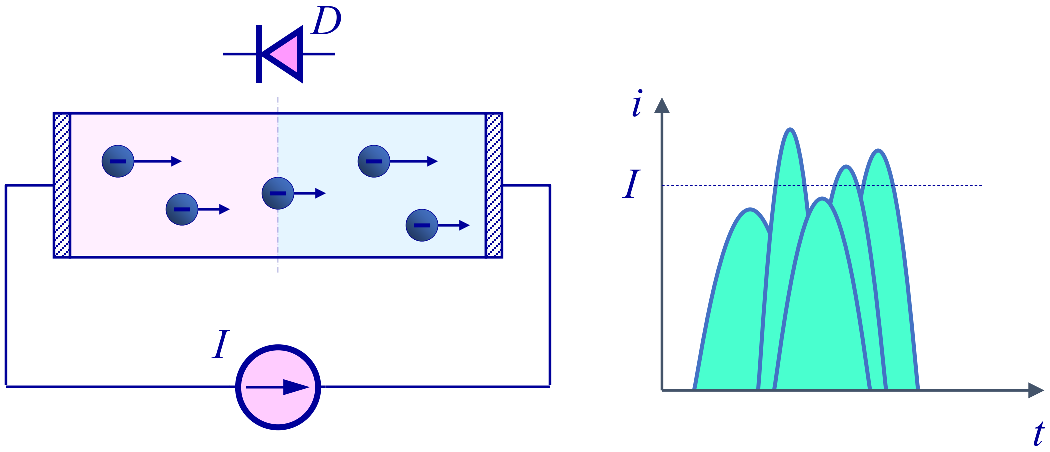
Q

-> WSS white noise, but in contrast to thermal noise, shot noise *cannot* be reduced by lowering the *temperature*!

S. Cova, "Sensors Signals and Noise, Course Slides", Politecnico Milano 2016

# 10.3.3 Shot Current

- Example: Shot Noise in a Diode




E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006 -> K. B. Klaassen, *Electronic measurement and instrumentation*, Cambridge University Press, 1996.

# 10.3.3 Shot Current in p-n Junction

- In a reverse-biased p-n junction, the shot current is a random sequence of statistically independent Dirac pulses $f(t)$

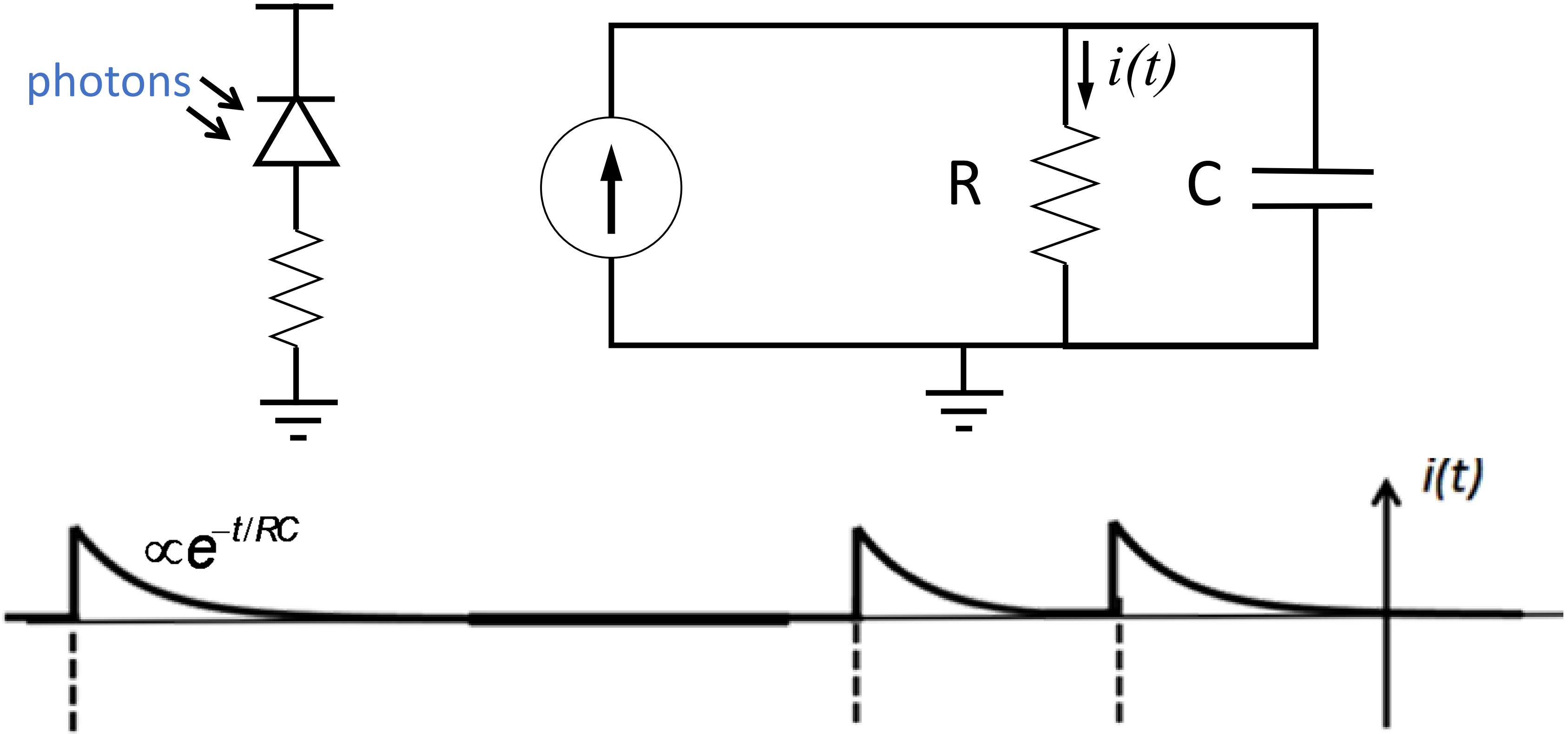
- It means that the probability for a pulse to occur is independent of the occurrence of other pulses
  - No history
  - $p(t)dt$ is the probability that a pulse starts in $[t, t + dt]$
  - We consider $p(t) = p$ (independent of $t$) and $p$ equals to $I/q$

*Q*

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

# 10.3.3 Shot Current in p-n Junction Diode (photocurrent)

- In reality - macroscopically - the white noise is filtered by the parasitics of the junction, thus the Dirac pulses are visible as series of exponential pulses:




📖 E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

# 10.3.3 Shot Current in p-n Junction Diode (photocurrent)

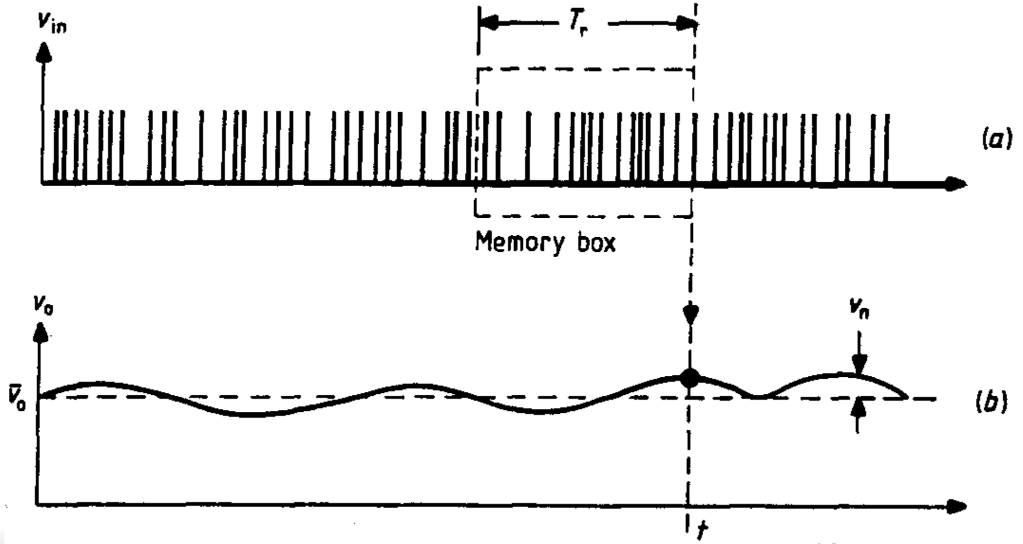
- Photons produce electron-hole pairs with a small probability, known as Quantum Efficiency (QE) **Q**

- QE = percentage of photons impinging on a diode that produce a charge carrier

→ Since the charge of the electron is q = 1.6×10$^{-19}$ Coulomb, and remembering that $C = {}^{Q}/_{V} \rightarrow V = {}^{Q}/_{C}$, the voltage seen across the resistor is at most q/C = 1.6 µV (assuming C = 100 fF and no current goes through R initially)

- Thus, macroscopically it will be seen as continuous noise - the individual fluctuations are not visible anymore, all the more so as the photon flux increases.

*Alternative derivation: count the number of quanta within a square time window of width T (equivalent to an integration over time)*





M. Van Exter, "Noise and Signal Processing", Univ. Leiden, 2003

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

# 10.3.3 Shot Noise (photocurrent)

S

- Example: let us compute the photocurrent in a p-n junction for a radiating power of 100nW of red light ($\lambda = 612nm$) onto the photodiode

- The photon flux is:

$$PF_D = \frac{P_D}{hc/\lambda} = \frac{100 \text{ nW}}{3.23 \times 10^{-19}\text{J}} = 3.1 \times 10^{11} \text{ photons/s}$$

$$I = QE \cdot PF_D \cdot q = 35 \text{ nA}$$

$$\text{where } h\text{: Planck's Constant} = 6.626 \times 10^{-34} \text{ J} \cdot s$$

$$\text{Assume } QE = 0.7$$

E. Charbon, “Imaging Sensors – ET 4390 Course Slides”, Delft 2016

# 10.3.3 Shot Current (photocurrent) vs. Poisson Statistics

- The noise is given by the fact that ***photons*** don't reach the diode uniformly, but follow a Poisson statistics: the probability that $k$ photons reach the diode in a given time interval $\Delta t$ is ($\lambda$ = rate of photon arrivals = mean number of photons reaching the diode in the time interval $\Delta t$):

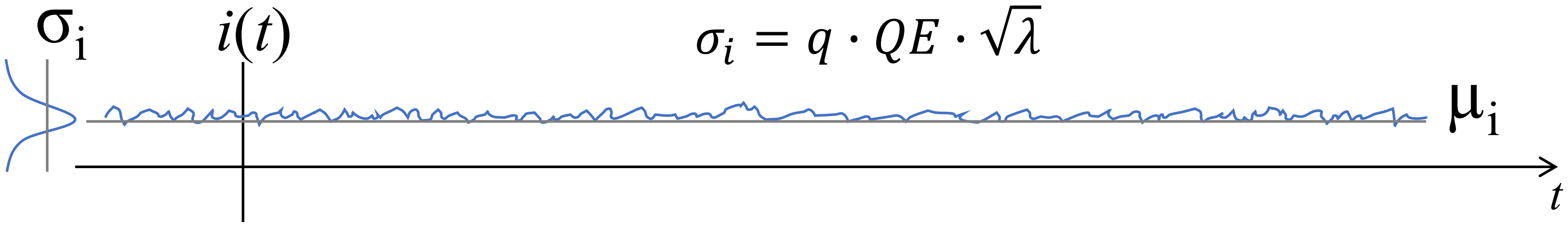
$$P\{X = k\} = \frac{\lambda^k e^{-\lambda}}{k!}$$

- If each carrier is generated by a photon in the photodiode, then the **photocurrent** also follows Poissonian statistics:

$$i(t) = q \cdot QE \cdot k(t)$$

$$E\{i(t)\} = \mu_i = q \cdot QE \cdot \lambda$$

$$\sigma_i = q \cdot QE \cdot \sqrt{\lambda}$$

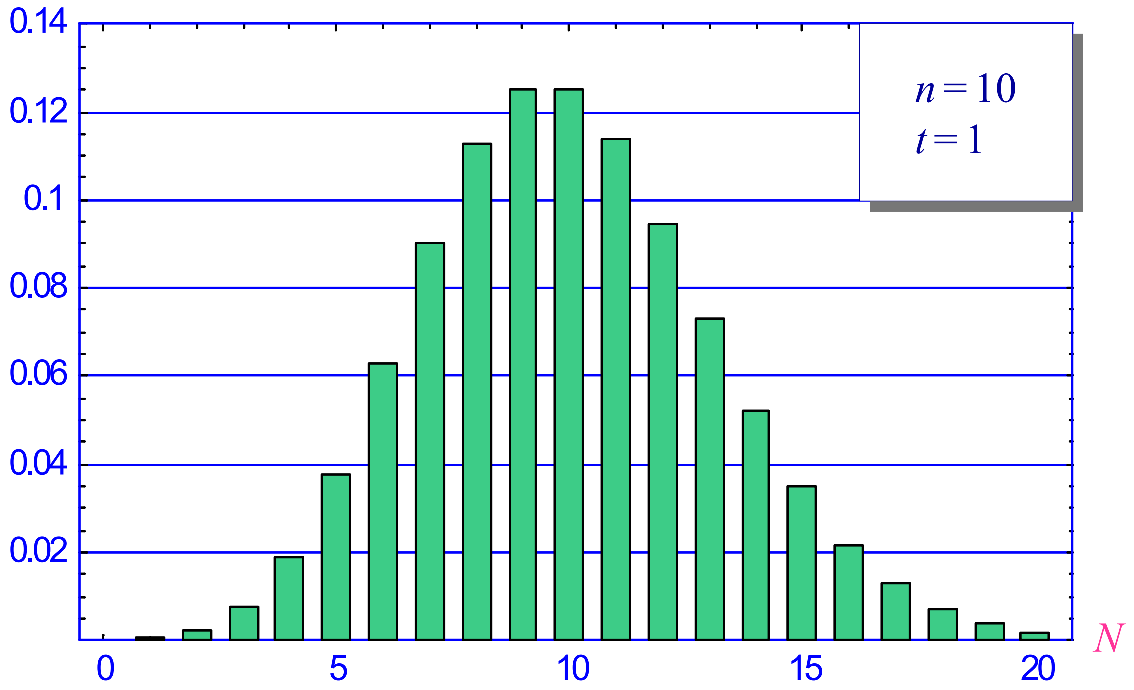


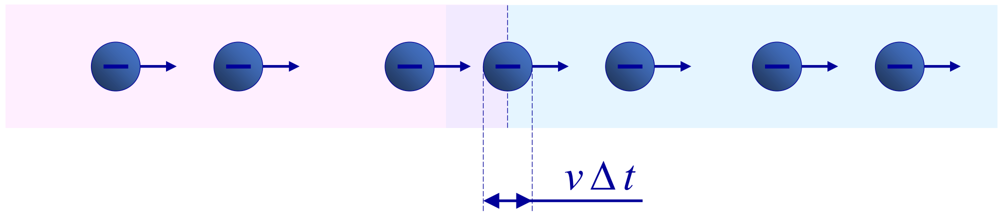

- The current will (also) be a random process, i.e. a collection of RVs with Poisson statistics

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

# 10.3.3 Shot Noise: Poisson Probability Distribution

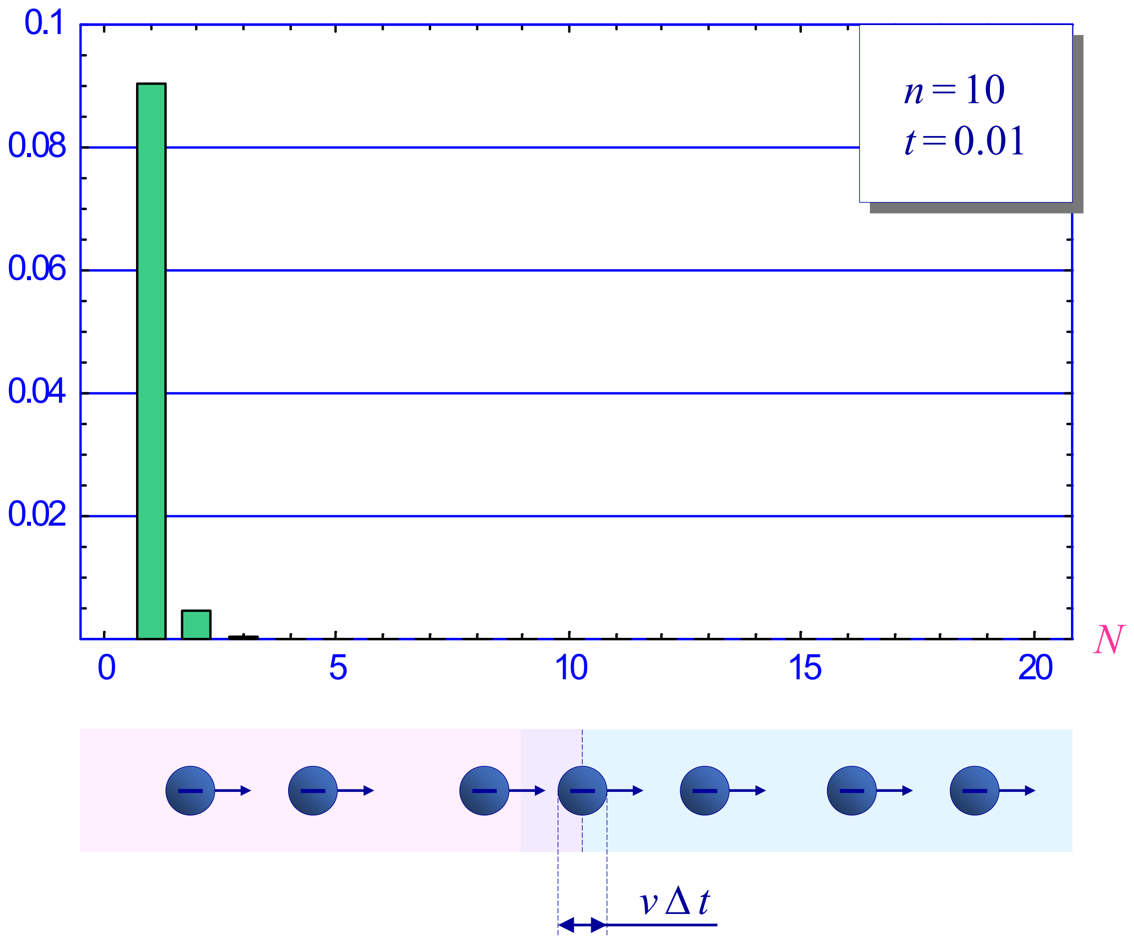
$$P_n(t) = \frac{(nt)^n}{n!} e^{-nt}$$



# 10.3.3 Shot Noise: Poisson Probability Distribution

$$P_n(t) = \frac{(nt)^n}{n!} e^{-nt}$$

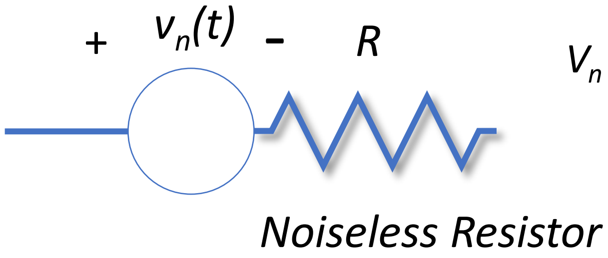


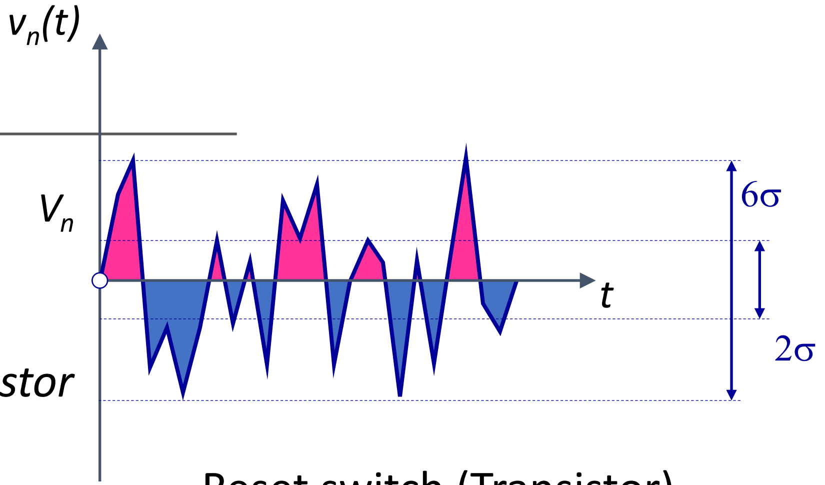
# 10.3.3 Shot Noise - Summary

- Through a *p-n* junction (or any other potential barrier), the electrons are transmitted *randomly* and *independently* of each other. Thus the transfer of electrons can be described by Poisson statistics.
- Shot noise is absent in a *macroscopic*, metallic resistor because the ubiquitous inelastic electron-phonon scattering smoothes out current fluctuations that result from the discreteness of the electrons, leaving only thermal noise.
- Shot noise *does* exist in *mesoscopic* (nm) resistors, although at lower levels than in a diode junction. For these devices the length of the conductor is short enough for the electron to become correlated, a result of the Pauli exclusion principle.
  - In this case the electrons are no longer transmitted randomly, but according to *sub-Poissonian* statistics.

M. de Jong, https://www.lorentz.leidenuniv.nl/beenakker/beenakkr/mesoscopics/topics/noise/noise.html (published in *Physics World, August 1996, page 22*)

# Take-home Messages/W4-2

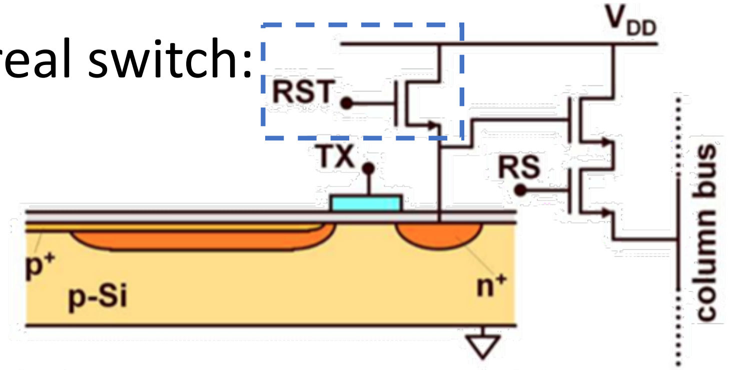


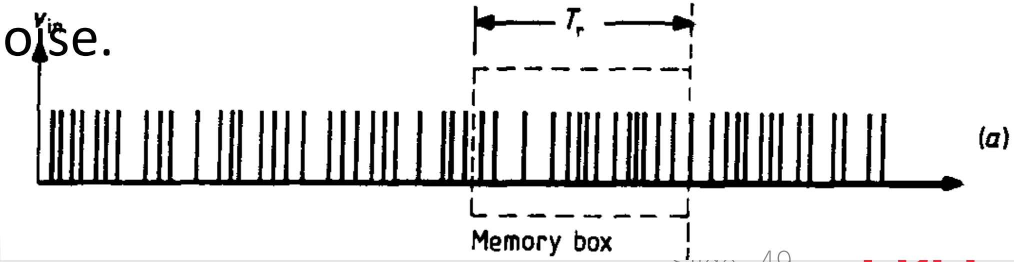

- *Noise Sources:*
  - Thermal noise, e.g. in resistors.
    - White noise. Bilateral PSD: $2kTR$.

Reset switch (Transistor) schematic

  - kTC noise (thermal noise on capacitors), e.g. when opening a real switch:
    - $\overline{V_0^2} = \frac{kT}{C}$




  - Shot (or Poisson noise), e.g. current flow through a potential barrier, or in photon-counting devices. Bilateral PSD: $S$=$qI$, WSS white noise.
  - NB: thermal and shot noise are irreducible.



# 10.3.4 Flicker (1/f) Noise

- Thermal noise and shot noise are irreducible (ever present) forms of noise. They define the minimum noise level or the 'noise floor'. Many devices generate additional or *excess noise.*
- The most general type of excess noise is $1/f$ or flicker noise. This noise has approximately $1/f$ power spectrum (equal power per decade of frequency) and is sometimes also called pink noise. ***Ex***
- $1/f$ noise is usually related to the fluctuations of the device properties caused, for example, by electric current in resistors and semiconductor devices.
- Curiously enough, $1/f$ noise is present in nature in unexpected places,
  - the speed of ocean currents
  - the flow of traffic on an expressway
  - the loudness of a piece of classical music versus time,
  - and the flow of sand in an hourglass.
- *No unifying principle* has been found for all the $1/f$ noise sources.

E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006 -> P. Horowitz and W. Hill, *The art of electronics*, Second Edition, Cambridge University Press, 1989.

# 10.3.4 Flicker Noise

- In electrical and electronic devices, flicker noise occurs only when electric current is flowing.
- In semiconductors, flicker noise usually arises due to traps, where the carriers that would normally constitute dc current flow are held for some time and then released.
- Although bipolar, JFET, and MOSFET transistors have flicker noise, it is a significant noise source in MOS transistors, whereas it can often be ignored in bipolar transistors.
- Summarising, it is observed in all electronic devices:
  - Strong in MOSFETs
  - Moderate in Bipolar transistors BJTs
  - Moderate in carbon resistors
  - Ultra-weak in metal-film resistors

📖 E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006 -> R. B. Northrop, *Introduction to instrumentation and measurements*, second edition, CRC Press,2005 & D. A. Jones and K. Martin, *Analog integrated circuit design*, John Wiley & Sons, 1997.

📖 E. Charbon, “Imaging Sensors – ET 4390 Course Slides”, Delft 2016; S. Cova, “*Sensors Signals and Noise – Course Slides*”, Politecnico di Milano 2016

# 10.3.4 Flicker Noise

- $1/f$ noise arises from physical processes that generate a random superposition of elementary pulses with random pulse duration ranging from **very short to very long**

Q

- E.g. in MOSFETs $1/f$ noise is due to:
  - Carriers traveling in the conduction channel are captured by local trap
  - trapped carriers are later released by the level with a random delay
  - the level lifetime (=mean delay) strongly depends on how distant from the silicon surface (= from the conduction channel) is the level in the oxide
  - trap levels are distributed from very near to very far from silicon, lifetimes are correspondingly distributed from very short to very long

Example: *distributed trapping model* (superposition of uniform exponential relaxation processes = superposition of Lorentzian PSDs, corresponding to RTS behaviours, with uniform distribution of lifetimes)…

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016; S. Cova, "*Sensors Signals and Noise – Course Slides*", Politecnico di Milano 2016

# 10.3.4 Flicker Noise

- Basic distinction between $1/f$ and white noise: time span of interdependence between samples

  - for white noise: samples are uncorrelated even at short time distance
  - for $1/f$ noise: samples are strongly correlated even at long time distance

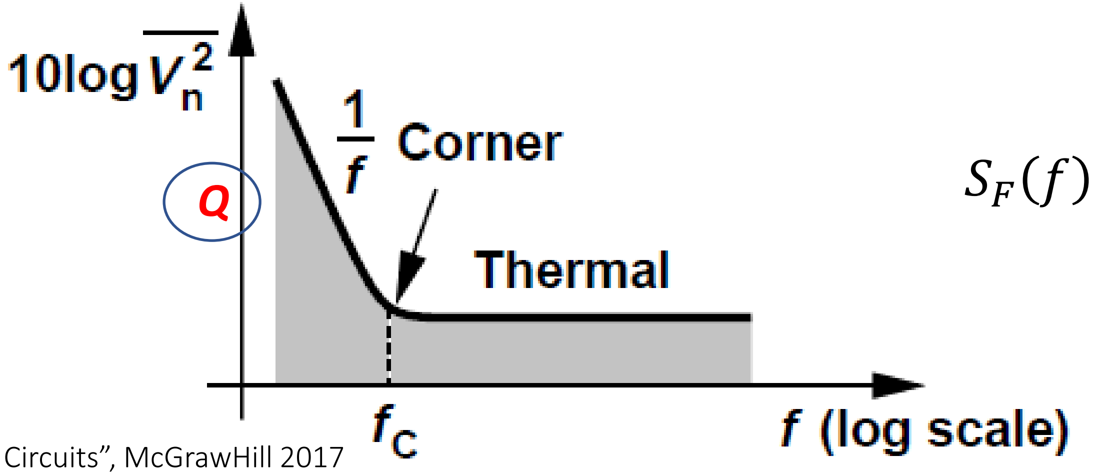
- Often the relation is more complex:

$$S_F(\omega) \propto \frac{1}{|f|^{\alpha}},\ 0.8 < \alpha < 1.2$$

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016; S. Cova, "*Sensors Signals and Noise – Course Slides*", Politecnico di Milano 2016

# 10.3.4 Flicker Noise

- At low frequencies, the flicker noise power approaches infinity
- At very slow rates, flicker noise becomes indistinguishable from thermal drift or aging of devices
- Noise component below the lowest frequency in the signal of interest does not corrupt it significantly
- Intersection point of thermal noise and flicker noise spectral densities is called "corner frequency" $f_C$, usually between 10 Hz and 1 MHz




$$S_F(f) = \frac{P}{|f|}, \quad \overline{n_F^2} = \int_0^\infty \frac{P}{|f|} df$$

(spectral density, unilateral; noise power)

B. Razavi, "Design of Analog CMOS Integrated Circuits", McGrawHill 2017

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016; S. Cova, "*Sensors Signals and Noise – Course Slides*", Politecnico di Milano 2016

# 10.3.4 Flicker Noise

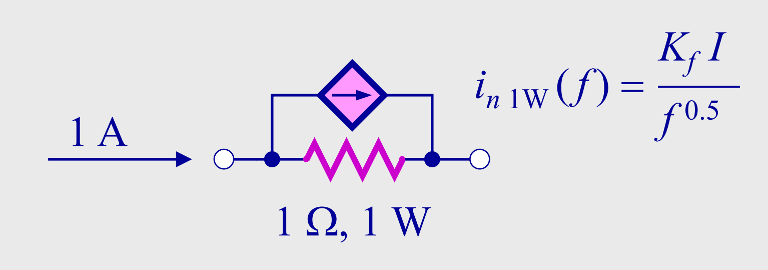
S

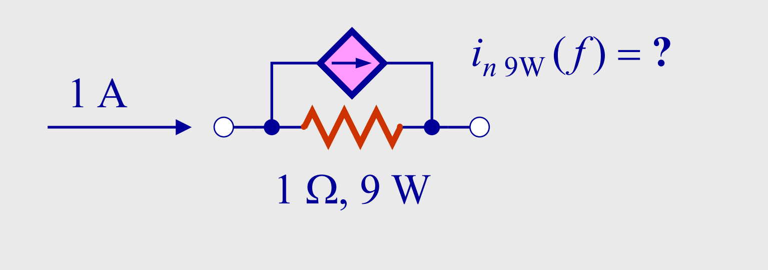
- [Resistors] Flicker noise is directly proportional to the dc (or average) current flowing through the device ($K_f$ = constant depending on type of material)

- The spectral power density of $1/f$ noise in resistors is in inverse proportion to their power dissipating rating.

- This is so, because the resistor current density decreases with the square root of its power dissipating rating.

- Exercise: compare $1/f$ noise in 1 Ω, 1 W and 1 Ω, 9 W resistors for the same 1 A dc current…

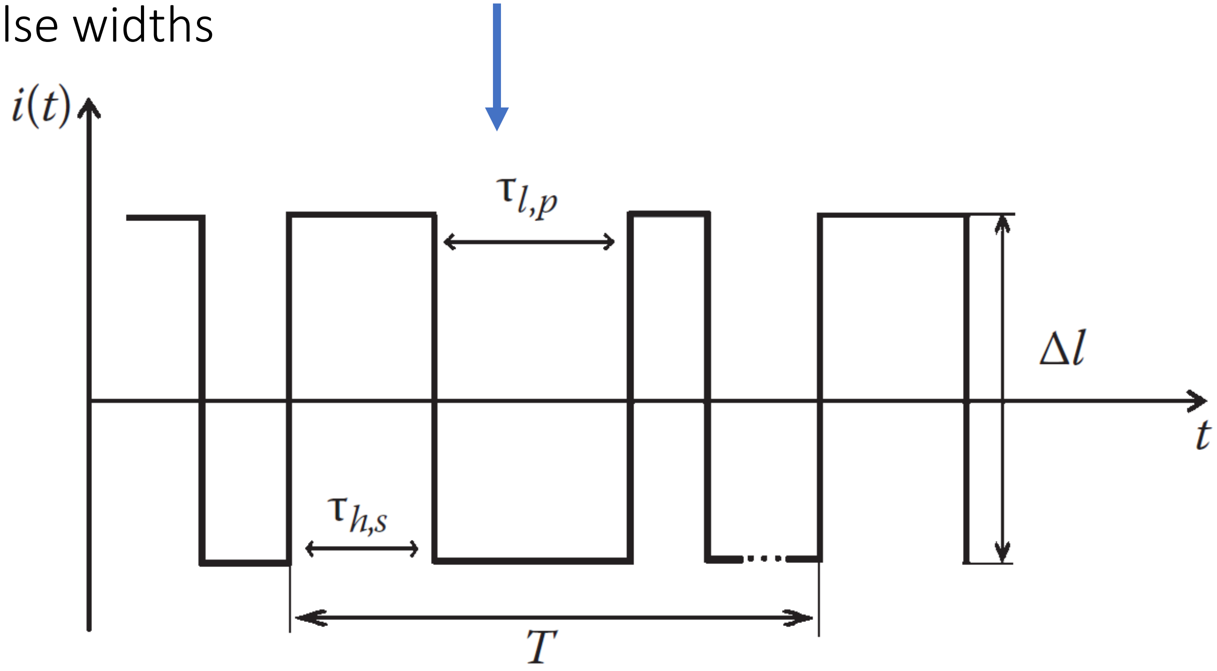



E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006 -> R. B. Northrop, *Introduction to instrumentation and measurements*, second edition, CRC Press,2005 & D. A. Jones and K. Martin, *Analog integrated circuit design*, John Wiley & Sons, 1997.

# 10.3.5 RTS Noise

- Burst noise is another type of noise at low frequencies.
- Recently, this noise was described as RTS (Random Telegraph Signal) noise.
- With given biasing condition of a device the magnitude of pulses is constant, but the switching time is random.
- The burst noise looks, on an oscilloscope, like a square wave with the constant magnitude, but with random pulse widths




A. Konczakowska & B. M. Wilamowski, *Noise in Semiconductor Devices*, in *Fundamentals of Industrial Electronics*, Ch. 11, 2011

# 10.3.5 RTS Noise

- This kind of noise is due to generation/recombination effects and trapping in semiconductors.

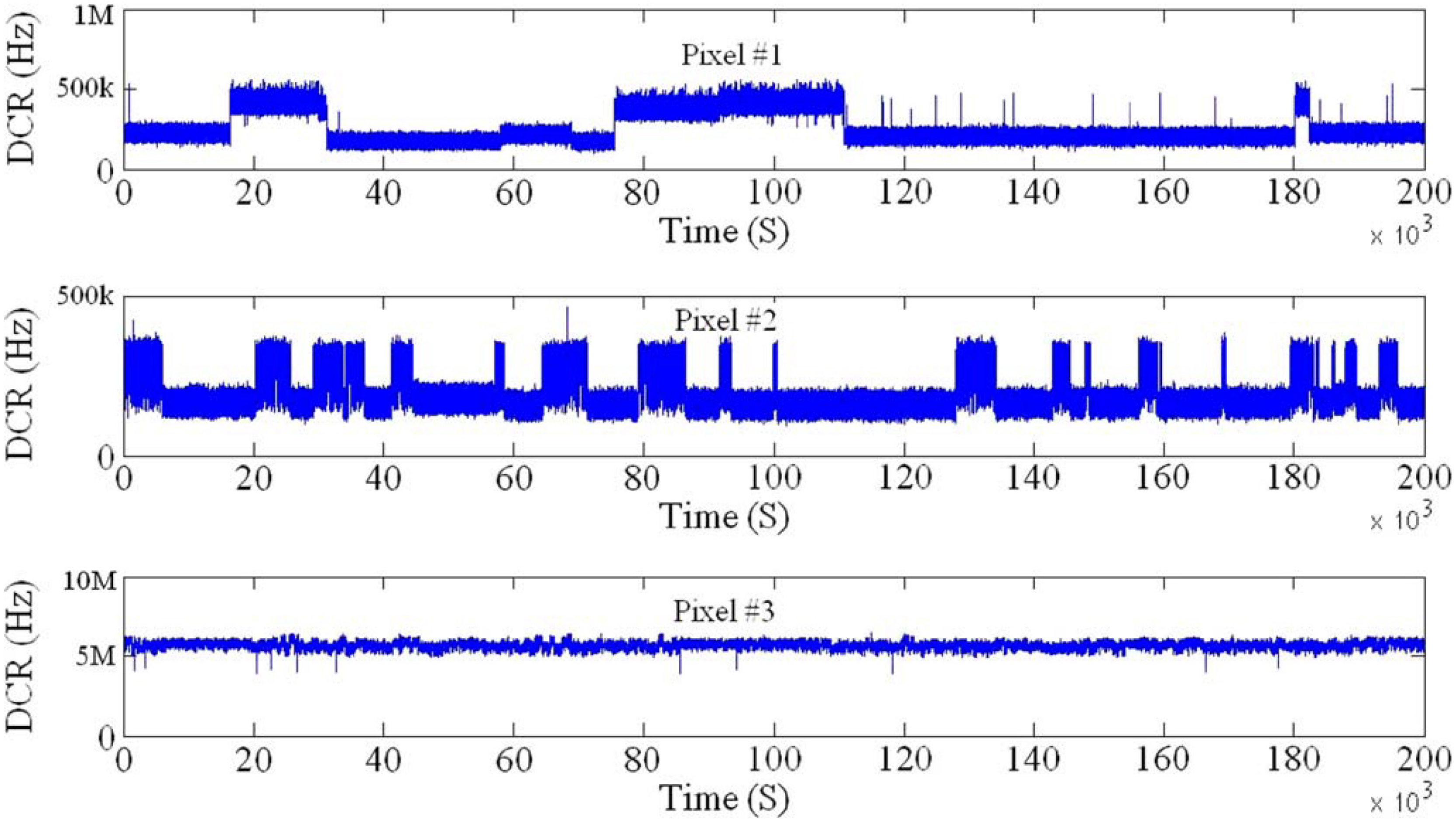
- Telegraph/Popcorn Noise is bias and frequency dependent

- *Lorentzian spectrum*: The Noise spectral density function of the RTS noise has a similar form like generation-recombination noise ($f_{RTS}$ = RTS noise corner frequency, below this frequency spectrum the RTS noise is flat)

$$S_{RTS}(f) = C\,\frac{4(\Delta I)^2}{1 + \left({}^{2\pi f}/_{f_{RTS}}\right)^2}$$

📖 A. Konczakowska & B. M. Wilamowski, *Noise in Semiconductor Devices*, in *Fundamentals of Industrial Electronics*, Ch. 11, 2011

# 10.3.5 RTS Noise – Example, non-stationary




***Q***

(radiation tolerance characterization, aging emulation, dose-rate effects)

M. A. Karami et al., *Random Telegraph Signal in Single-Photon Avalanche Diodes*, International Image Sensor Workshop, Bergen, 2009

# 10.3.5 RTS Noise

- This noise has significant effect at low frequencies.
- It is a function of temperature, induced mechanical stress, and also radiation.
- In audio amplifiers, the burst noise sounds as random shots, which are similar to the sound associated with making popcorn.
- Obviously, BJTs with large burst noise must not be used in audio amplifiers and in other analog circuitry.
- It is now assumed that devices fabricated with well-developed and established technologies do not generate RTS noise.
- This is unfortunately not true for modern nanotransistors and devices fabricated with other than silicon materials.

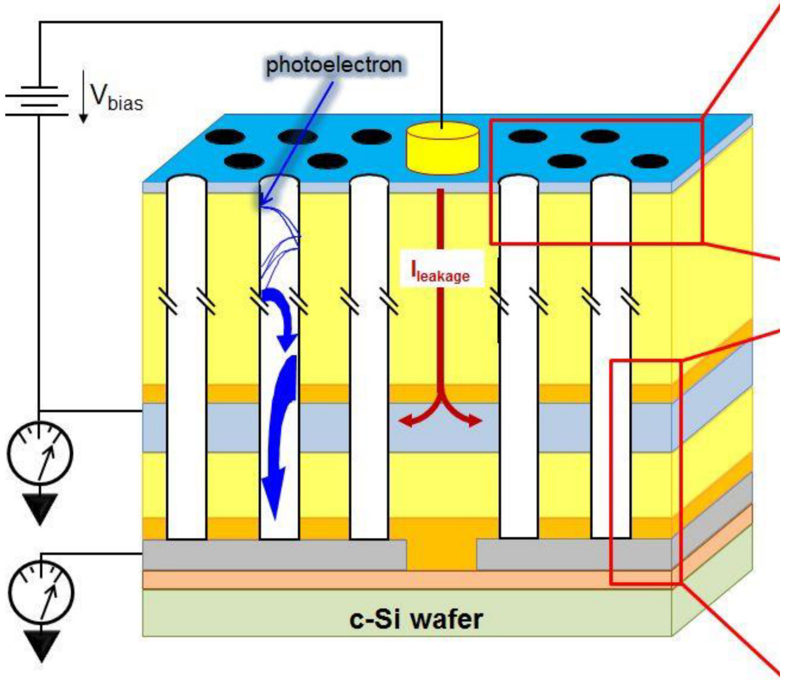
A. Konczakowska & B. M. Wilamowski, *Noise in Semiconductor Devices*, in *Fundamentals of Industrial Electronics*, Ch. 11, 2011

# 10.3.6 Other types of noise

- Quantization Noise
- Avalanche Noise
- Intermodulation Noise
- Crosstalk Noise
- Transit Time Noise

| | | | | |
|---|---|---|---|---|
| -0.001 | 0.007 | 0.007 | 0.006 | -0.001 |
| 0.004 | 0.034 | 0.057 | 0.032 | -0.003 |
| 0.000 | 0.056 | 100 | 0.059 | 0.001 |
| 0.001 | 0.030 | 0.053 | 0.029 | 0.003 |
| -0.001 | 0.006 | 0.004 | 0.000 | 0.001 |

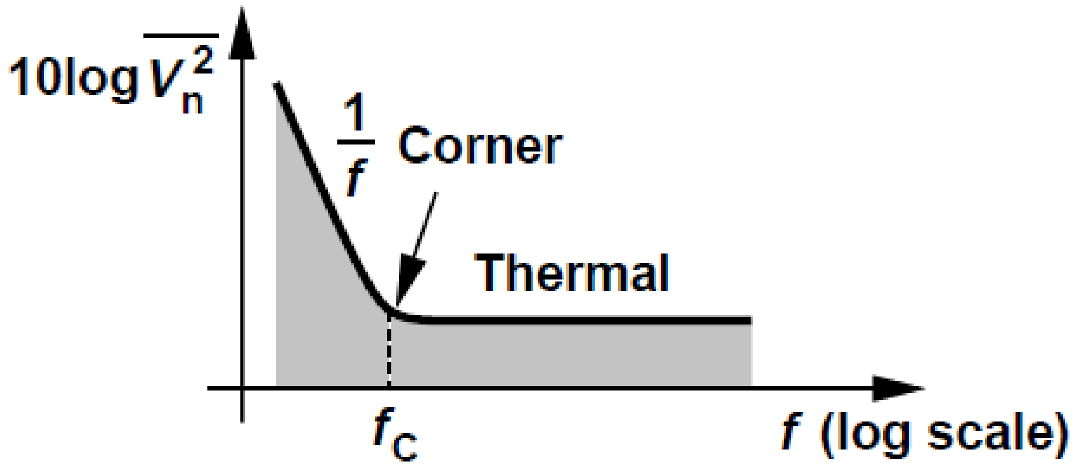
SwissSPAD2 Crosstalk Noise example

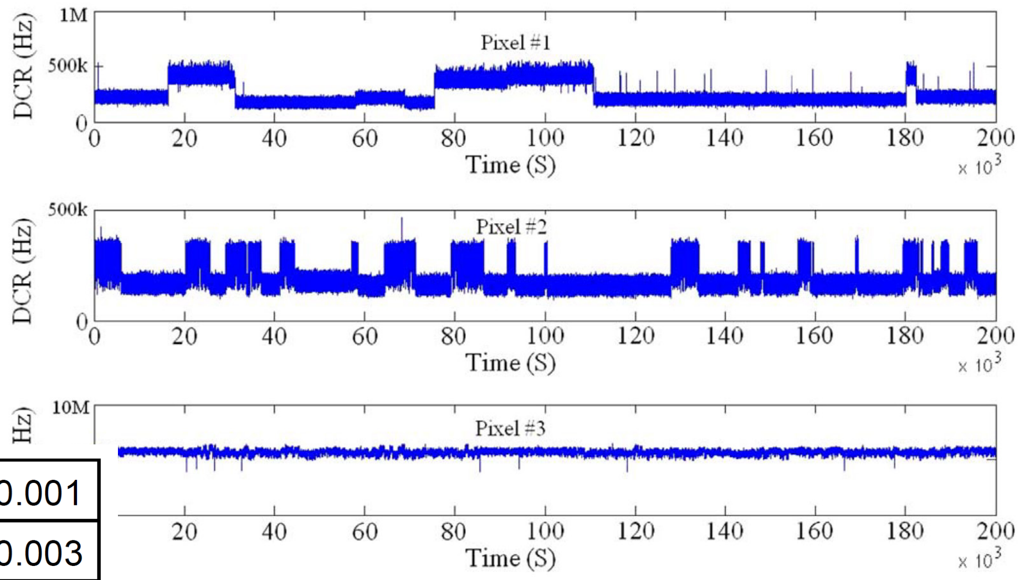



Transit Time Noise example

# Take-home Messages/W4-3

- *Noise Sources:*
  - Flicker (1/f) noise
    - $S_F(\omega) \propto \frac{1}{|f|^{\alpha}}, 0.8 < \ \alpha < 1.2$
  - RTS noise:
    - $S_{RTS}(f) = C \frac{4(\Delta I)^2}{1+\left({}^{2\pi f}/_{f_{RTS}}\right)^2}$
  - Other noise sources




| | | | | |
|---|---|---|---|---|
| -0.001 | 0.007 | 0.007 | 0.006 | -0.001 |
| 0.004 | 0.034 | 0.057 | 0.032 | -0.003 |
| 0.000 | 0.056 | 100 | 0.059 | 0.001 |
| 0.001 | 0.030 | 0.053 | 0.029 | 0.003 |
| -0.001 | 0.006 | 0.004 | 0.000 | 0.001 |

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- Bedirhan Ilik (TA 2019)
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- Estelle Labonne
- Akira Matsuzawa
- Mo Beygi, Samira Frei, PV-lab, EPFL NE

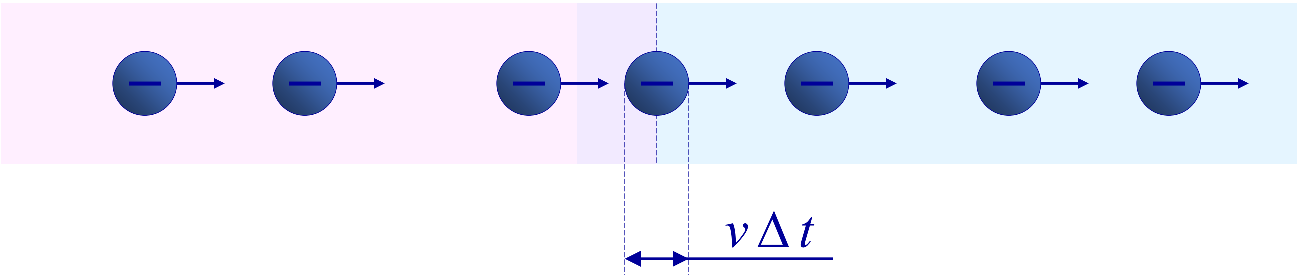
# Appendix A - 10.3.3 Shot Noise vs Poisson Statistics

[Derivation of Poissonian statistics]

- Start from defining $n$ as the average number of electrons crossing the p-n junction of a diode during one second, hence, the average electron current $I = qn$.
- Assume that the probability of two or more electrons crossing simultaneously is negligibly small, $P_{>1}(dt) = 0$.
- This allows us to define the probability that an electron crosses the junction in the time interval

$$dt = (t, t + dt) \quad \text{as} \quad P_1(dt) = n\,dt$$

(dt is approaching the time taken for an electron to travel across the junction, < 1 ns).

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# 10.3.3 Shot Noise vs Poisson Statistics

Next, we derive the probability that no electrons crosses the junction in the time interval $(0, t + d\,t)$:

$$P_0(t + d\,t) = P_0(t)\,P_0(d\,t) = P_0(t)\,[1 - P_1(d\,t)] = P_0(t) - P_0(t)\,n\,d\,t.$$

This yields:

$$\frac{dP_0}{dt} = -nP_0$$

with the obvious initiate state $P_0(0) = 1$.

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# 10.3.3 Shot Noise vs Poisson Statistics

The probability that an electron crosses the junction in the time interval $(0, t + \mathrm{d}\, t)$

$$P_1(t + d\,t) = P_1(t)\,P_0(d\,t) + P_0(t)\,P_1(d\,t)$$

$$= P_1(t)\,(1 - n\,d\,t) + P_0(t)\,n\,d\,t\,.$$

This yields

$$\frac{dP_1}{dt} = -nP_1 + nP_0$$

with the obvious initiate state $P_1(0) = 0$.

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# 10.3.3 Shot Noise vs Poisson Statistics

- In the same way, one can obtain the probability of *N* electrons crossing the junction:

$$\begin{cases} \dfrac{dP_n}{dt} = -nP_n + nP_{n-1} \\ P_n(0) = 0 \end{cases}$$

By substitution, one can verify that

$$P_n(t) = \frac{(nt)^n}{n!} e^{-nt},$$

which corresponds to the Poisson probability distribution.

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